# LANGRANGIAN MECANICS

1) Generalised co-ordinate :- To describe the configuration of a system we select smallest possible no. of variables K/a Generalised co-ordinates.

The Generalised co-ordinate is represented by $q_1, q_2, q_3 \ldots q_n$. If there are 'K' contrained of motion the generalised co-ordinate is reduced to $(n-K)$ for the system of n particle. The position of ith particle in the form of Generalised co-ordinate is given by

$$x_i = x_i (q_1, q_2, q_3 \ldots q_n, t)$$
$$y_i = y_i (q_1, q_2, q_3 \ldots q_n t).$$
$$z_i = z_i (q_1, q_2, q_3 \ldots q_n t)$$
$$\Rightarrow r_i = r_i (q_1, q_2, q_3 \ldots q_n, t)$$

Hence a genaralised co-ordinate is defined as the independent co-ordinate sufficient to specify the configuration of dynamical system

2) Principle of Virtual Work :- The total workdone by the particle of system in equilibrium for small disp. $\delta r_i$ due to the force $F_i$ acting upon ith particle is equal to zero

$$\boxed{\sum_i F_i \delta r_i = 0}$$

3) D'Alembert's Principle :- The D'Alemberts Principle depends upon the Principle of Virtual work. The Virtual Work states the total workdone by the system of particle for ith particle due to force $F_i$ small displacement $\delta r_i$ for ith particle due to force $F_i$ is equal to zero

$$\sum F_i \delta r_i = 0$$

According to D'Alemberts, the system of Particle will be in equilibrium under the action force $F_i$ and reactional force $\dot{p}_i$

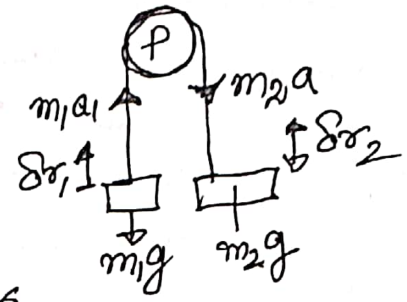
$$\boxed{\sum (F_i - \dot{p}_i) \delta r_i = 0}$$

This is K/a D'Alemberts Principle.

# Application of D'Alembert's Principle

① At Wood Mechine :-

Let us consider At Wood mechine having masses $m_1$ & $m_2$ acceleration $a_1$ and $a_2$ & their displacement in time $\delta t$ are $\delta r_1$ & $\delta r_2$ respectively.




Applying D'Alembert's principle

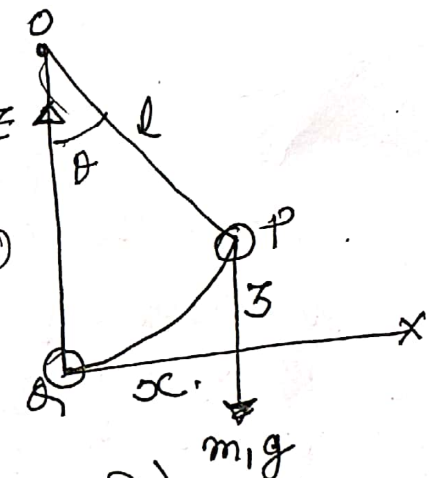
$$(m_1 g_1 - m_1 a_1)\delta r_1 + (m_2 g - m_2 a_2)\delta r_2 = 0$$

Now $\delta r_2 = -\delta r_1$ & $a_2 = -a_1$

$$\therefore (m_1 g - m_1 a_1)\delta r_1 - (m_2 g + m_2 a_1)\delta r_1 = 0$$

$$\Rightarrow \boxed{a_1 = \left(\frac{m_1 - m_2}{m_1 + m_2}\right) g.}$$

② Simple pendulum :- Let us consider a simple pendulum of length $l$ & mass of bob is $m$. The vertical displacement of bob along $x$ and $z$-axis are $\delta x$ & $\delta z$. The applied force along these axis are "X" and Z




$$(X - m\ddot{x})\delta x + (Z - m\ddot{z})\delta z = 0 \quad \text{—(A)}$$

Now $Z = -mg$ & $l^2 = z^2 + x^2$

$$0 = 2z\delta z + 2x\delta x$$

$$\Rightarrow \delta z = -\frac{x\delta x}{z}$$

From equ$^n$(A), $0 - m\ddot{x}\delta x + [-mg - m\ddot{z}]\left(-\frac{x\delta x}{z}\right) = 0$

$$\text{or, } \ddot{x} = (g + \ddot{z})\frac{x}{z}.$$

Hence the accel$^n$ is directly proportional to the displacement. The motion will be S.H.M. The time period of S.H.M is given by.

$$T = 2\pi\sqrt{\text{disp}/\text{accel}^n}.$$

$$= 2\pi\sqrt{x/\ddot{x}} =$$

$$\boxed{T = 2\pi\sqrt{\frac{l}{g}}}$$

$\because \ddot{z} = 0$ & $z = l$.