PHYSICS B.Sc Part-III (H) Paper-V Gr-C

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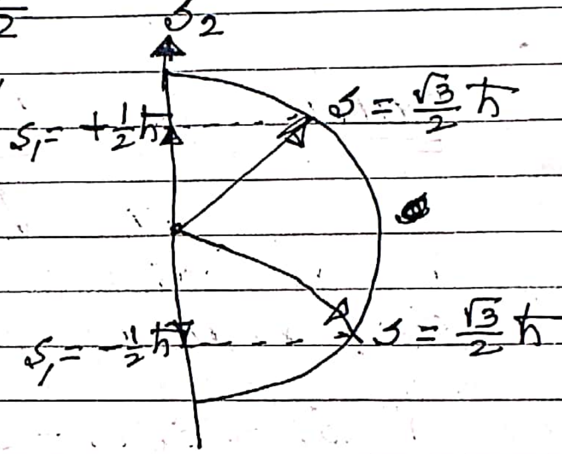
SPIN HALF ANGULAR MOMENTUM

The electron spins about an axis passing through its centre of mass. Therefore it has an internal or intrinsic angular momentum and a magnetic moment associated with spin rotation. The spin angular momentum of electron is given by

$$S = \sqrt{s(s+1)}\,\hbar = \sqrt{\tfrac{1}{2}\left(\tfrac{1}{2}+1\right)}\,\hbar$$

Where s is spin quantum ~~to~~ number and l is orbital quantum number. It has only one value i.e $\frac{1}{2}$



In the presence of external magnetic field, the component of the angular momentum is given by

$$S_z = m_z\hbar$$
$$= \pm\frac{1}{2}\hbar$$

Where m_z is called spin magnetic quantum number.

The magnetic moment associated with the spin angular momentum is given by.

$$\vec{\mu}_s = 2\frac{e}{2m}\sqrt{s(s+1)}\,\hbar$$

$$= 2\,\mu_B\sqrt{s(s+1)}$$

Where μ_B = Bohr Magneton $= \frac{e\hbar}{2m}$

The z-component of magnetic ~~field~~ moment

$$(\mu_s)_z = 2\frac{e}{2m} S_z = 2\frac{e}{2m_z} m_z(\hbar$$

$$= \pm \frac{e}{2m}\hbar \left[\because m_z = \pm \tfrac{1}{2}\right]$$

Thus $(\mu_s)_z$ has only two values $\frac{e}{2m}\hbar$ and $-\frac{e}{2m}\hbar$ has been verified by Stern - Gerlach experiment.

Pauli's Spin Matrics

The spin operator S_x, S_y, S_z are associated with the component of spin angular momentum as:

$$[S_x, S_y] = S_x S_y - S_y S_x = i\hbar S_z$$

$$[S_y, S_z] = (S_y S_z - S_z S_y) = i\hbar S_x$$

$$[S_z, S_x] = (S_z S_x - S_x S_z) = i\hbar S_y. \quad \text{--- (1)}$$

If we consider the case of spin for electron each of operator must have just two eigen value $\frac{1}{2}\hbar$ & $-\frac{1}{2}\hbar$

$$\therefore S_x = \frac{1}{2}\hbar\sigma_x,\ S_y = \frac{1}{2}\hbar\sigma_y \ \&\ S_z = \frac{1}{2}\hbar\sigma_z \quad \text{--- (2)}$$

Where $\sigma_x, \sigma_y, \sigma_z$ are arbitrary operators

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The operator $\sigma_x^2, \sigma_y^2, \sigma_z^2$ must have only its eigen value one i.e. these are unit operator

$$\sigma_x^2 = \sigma_y^2 = \sigma_z^2 = 1$$

The commutation rules are

$$[\sigma_x, \sigma_y] = [\sigma_x\sigma_y - \sigma_y\sigma_x] = 2i\sigma_z$$
$$[\sigma_y \sigma_z] = [\sigma_y\sigma_z - \sigma_z\sigma_y] = 2i\sigma_x$$
$$[\hat{\sigma}_z, \hat{\sigma}_x] = [\sigma_z\sigma_x - \sigma_x\sigma_z] = 2i\sigma_y \qquad (3)$$

Now $2i(\sigma_x\sigma_y - \sigma_y\sigma_x) = 2i\cdot\sigma_x\sigma_y + \sigma_y\cdot 2i\sigma_x$

$$= (\sigma_y\sigma_z - \sigma_z\sigma_y)\sigma_y + \sigma_y[\sigma_y\sigma_z - \sigma_z\sigma_y]$$
$$= 0$$

$\Rightarrow \sigma_x\sigma_y + \sigma_y\sigma_x = 0$

Similarly $\sigma_y\sigma_z + \sigma_z\sigma_y = 0$

$$\sigma_z\sigma_x + \sigma_x\sigma_z = 0 \qquad (4)$$

From equ^n (3) & (4) We get

$$\sigma_x\sigma_y = i\sigma_z$$
$$\sigma_y\sigma_z = i\sigma_x$$
$$\sigma_z\sigma_x = i\sigma_y \qquad (5)$$

Since each σ have two ~~value~~ eigen values so 2×2 matrix having its eigen value 1 & -1

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$$\therefore \sigma_z = \begin{vmatrix} 1 & 0 \\ 0 & -1 \end{vmatrix}$$

$$\sigma_x = \begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix}$$

$$\sigma_y = \begin{vmatrix} 0 & -i \\ -i & 0 \end{vmatrix}$$

Here $\sigma_x, \sigma_y, \sigma_z$ are called Pauli's spin matrices associated with the component of spin angular momentum

$$S_x = \frac{1}{2}\hbar \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$S_y = \frac{1}{2}\hbar \begin{bmatrix} 0 & -i \\ -i & 0 \end{bmatrix}$$

$$S_z = \frac{1}{2}\hbar \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$