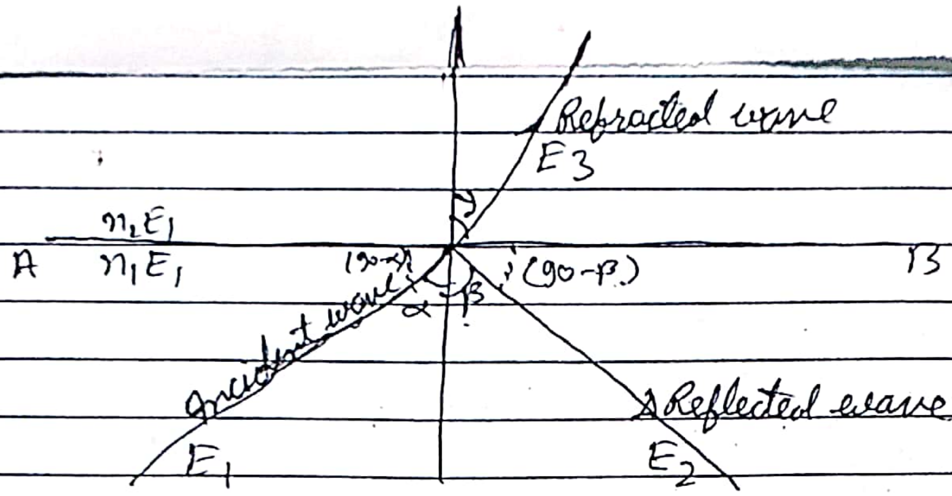
REFLECTION & REFRACTION OF ELECTRO MAGNETIC WAVE



Let us consider electromagnetic wave is allowed to incident upon AB which divide the medium of refractive indices n_1 & n_2 : ~~Let~~

Let us consider ~~electromagnetic~~ the electric vector of incident ray, refracted ray and reflected ray are E_1, E_2 & E_3

Let the wave factors of ~~incident~~ incident ray & reflected ray & refracted ray are K_1, K_2 & K_3

The eqn of incident wave, reflected wave and refracted wave are given by

$$\left.\begin{aligned} E_1 &= E_{01}\, e^{i(\omega t - K_1 r)} \\ E_2 &= E_{02}\, e^{i(\omega t - K_2 r)} \\ E_3 &= E_{03}\, e^{i(\omega t - K_3 r)} \end{aligned}\right\} \quad \text{—— } (A_1)$$

The scalar component of electric field along Y dirn

$$E_1 Y = A_1 Y e^{i(\omega_1 t - K_1 r)}$$

$$E_2 Y = A_2 Y e^{i(\omega_2 t - K_2 r)}$$

$$E_3 Y = A_3 Y e^{i(\omega_3 t - K_3 r)}$$

where A_1, A_2 & A_3 are the amplitude along incident ray, reflected ray & refracted ray.

Now $E_1 y + E_2 y = E_3 y$

$$A_1 y e^{i(\omega_1 t - K_1 r)} + A_2 y e^{i(\omega_2 t - K_2 r)} = A_3 y e^{i(\omega_3 t - K_3 r)} \quad \text{---(C)}$$

i.e.

Differentiating equ^n (3) w.r to time we get.

$$A_1 y i\omega_1 e^{i(\omega_1 t - K_1 r)} + A_2 y i\omega_2 e^{i(\omega_2 t - K_2 r)} = A_3 y e\omega_3^{i(\omega_3 t - K_3 r)} \quad \text{---(D)}$$

On solving equ^n (C) & (D),

$$A_1 y e^{i(\omega_1 t - K_1 r)} (\omega_3 - \omega_1) = A_2 y e^{i(\omega_2 t - K_2 r)} (\omega_2 - \omega_3) \quad \text{---(E)}$$

This equ^n is valid for $\omega_1 = \omega_2 = \omega_3$

From equ^n (C) we get.

$$A_1 y e^{-iK_1 r} + A_2 y e^{-iK_2 r} = A_3 y e^{-iK_3 r} \quad \text{---(F)}$$

Differentiating it w.r to r we get.

$$A_1 y i K e^{-iK_1 r} + A_2 y i K_2 e^{-iK_2 r} = A_3 y i K_3 e^{-iK_3 r} \quad \text{---(G)}$$

Taking dot product with $\vec{r}$ w.r.t.

$$\vec{K_1}\cdot\vec{r} A_1 y e^{-iK_1 r} + \vec{K_2}\cdot\vec{r} A_2 y e^{-iK_2 r} = \vec{K_3}\cdot\vec{r} A_3 y e^{-iK_3 r}$$

On solving ~~we~~ equ^n (G) & H we get.

$$A_1 y e^{-iK_1 r} (\vec{K_3}\cdot\vec{r} - \vec{K_1}\cdot\vec{r}) = A_2 y e^{-iK_2 r} (\vec{K_2}\cdot\vec{r} - \vec{K_3}\cdot\vec{r})$$

This equ^n is valid for

$$\vec{K_1}\cdot\vec{r} = K_1 r \cos(90-\alpha) = K_1 r \sin\alpha$$

$$\vec{K_2}\cdot\vec{r} = K_2 r \sin\beta$$

$$\vec{K_3}\cdot\vec{r} = K_3 r \sin\gamma$$

From equ^n (I) for reflection

$$\cancel{K_1} \; K_1 = K_2$$

$$\boxed{\alpha = \beta}$$

This is law of reflection

Again let the velocity of wave for incident ray and refracted ray are v_1 & v_2

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Using equ^n

$k_1 r \sin\alpha = k_2 r \sin\gamma$

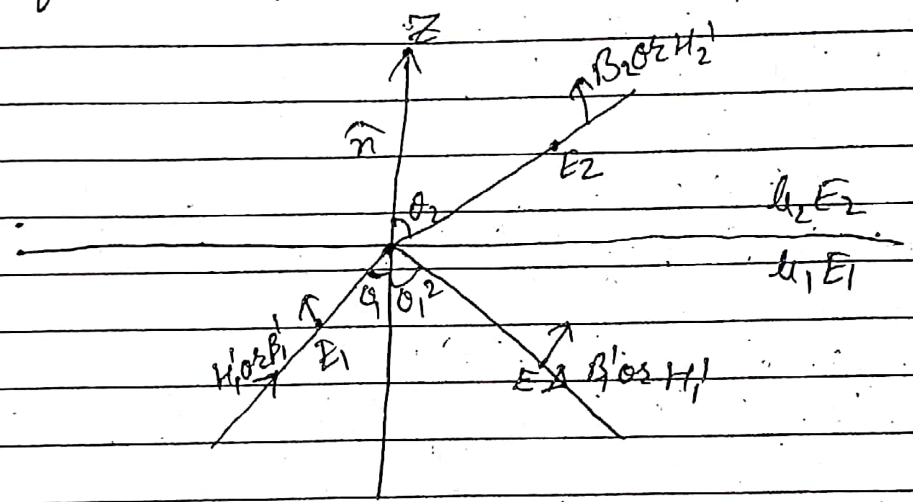
$\frac{\sin\alpha}{\sin\gamma} = \frac{k_2}{k_1} = \frac{v_1}{v_2} = \frac{n_2}{n_1}$

$\Rightarrow n_1 \sin\alpha = n_2 \sin\gamma$

This ~~is~~ is Snell's law of refraction

★ Fresnel equ^n of reflection and refraction :

The equ^n releating the amplitude of reflected and transmitted wave with those of incident wave are k/a fresnel's equ^n.



Let the electric vector E_1 and magnetic vector B_1 are $\perp^r$ upon the direction of propogation constant k_1

The equ^n of continuity of electric vector along the dir^n of reflected surface

$$EO_1 + EO_1' = EO_2 \quad \text{--- (1)}$$

Similarly the magnetic vector continuity along the z axis

$$H_1 t + H_1' t = H_2 t$$

$$HO_1 \cos\theta + HO_1' \cos\theta_1' = -HO_2 \cos\theta_2$$

$$(HO_1 - H'O_1) \cos\theta_1 = HO_2 \cos\theta_2 \quad \text{--- (2)}$$

$B_1 = \frac{k_1 \times E_1}{\omega_1} \Rightarrow H_1 = \frac{k_1 \times E_1}{\mu_1 \omega_1}$

(4)

or $H_1 = \frac{K_1 \hat{n}_1 \times E_1}{\mu_1 \omega_1}$

$K_1 = \frac{\omega_1}{V_1} = \sqrt{\varepsilon_1 \mu_1}\, \omega_1$

$\cancel{H_1 = \frac{\omega_1 \hat{n}_1 \times E_1}{V_1}}$

$H_1 = \frac{\sqrt{\varepsilon_1 \mu_1}\, \hat{n}_1 \times E_1}{\mu_1}$

$H_1 = \sqrt{\frac{\varepsilon_1}{\mu_1}}\, \hat{n}_1 E_1$

$$\left.\begin{aligned} HO_1 &= \sqrt{\frac{\varepsilon_1}{\mu_1}}\, EO_1 \\ H'O_1 &= \sqrt{\frac{\varepsilon_1}{\mu_1}}\, E'O_2 \\ HO_2 &= \sqrt{\frac{\varepsilon_2}{\mu_2}}\, EO_2 \end{aligned}\right\} \longrightarrow (3)$$

From eqn (2)

$\sqrt{\frac{\varepsilon_1}{\mu_1}}\left(EO_1 - EO_1'\right)\cos\theta_1 = \sqrt{\frac{\varepsilon_2}{\mu_2}}\, EO_2 \cos\theta_2$

using eqn (1)

$\sqrt{\frac{\varepsilon_1}{\mu_1}}\,(EO_1 - EO_1')\cos\theta_1 = \sqrt{\frac{\varepsilon_2}{\mu_2}}\,(EO_1 + EO_1')\cos\theta_2$

On solving we get

$\frac{EO_1 - E'O_1}{EO_1 + E'O} = \frac{\sqrt{\varepsilon_2/\mu_2}\cdot\cos\theta_2}{\sqrt{\varepsilon_1/\mu_1}\cos\theta_1}$

$\frac{2EO_1}{2EO_2} = \frac{\sqrt{\varepsilon_1/\mu_1}\cos\theta_1 + \sqrt{\varepsilon_2/\mu_2}\cos\theta_2}{\sqrt{\varepsilon_1/\mu_1}\cos\theta - \sqrt{\varepsilon_2/\mu_2}\cos\theta_2}$

Similarly eleminating EO_1' from (2) & (4)

$\sqrt{\frac{\varepsilon_1}{\mu_1}}\left[EO_1 - EO_2 + EO_1\right]\cos\theta_1 = \sqrt{\frac{\varepsilon_2}{\mu_2}}\, EO_2 \cos\theta_2$

$2EO_1 \sqrt{\frac{\varepsilon_1}{\mu_1}}\cos\theta_1 = EO_2\left[\sqrt{\frac{\varepsilon_2}{\mu_2}}\cos\theta_2 + \sqrt{\frac{\varepsilon_1}{\mu_1}}\cos\theta_1\right]$

$$\frac{E_{O_1}}{E_{O_2}} = \frac{\sqrt{\varepsilon_2/\mu_2}\cos\theta_2 + \sqrt{\varepsilon_1/\mu_1}\cos\theta_1}{2\sqrt{\varepsilon_1/\mu_1}\cos\theta_1} \quad \text{--- (6)}$$

These ~~of~~ eq^n (5) & (6) are K/a Fresnels eq^n for nonconducting medium.

$\therefore \mu_1 = \mu_2 = \mu_0$

then the refractive index of the medium

$$n_1 = \sqrt{\varepsilon_1/\varepsilon_0}$$

& $n_2 = \sqrt{\varepsilon_2/\varepsilon_0}$

$$\frac{n_2}{n_1} = \sqrt{\frac{\varepsilon_2}{\varepsilon_1}} \quad \text{--- (7)}$$

From eq^n (5) $\sqrt{\frac{\varepsilon_1}{\mu_0}}\cos\theta_1 + \sqrt{\frac{\varepsilon_2}{\mu_0}}\cos\theta_2$

$$\Rightarrow \frac{\cos\theta_1 + \sqrt{\varepsilon_2/\varepsilon_1}\cos\theta_2}{\cos\theta_1 - \sqrt{\varepsilon_2/\varepsilon_1}\cos\theta_2}$$

$$\frac{E_{O_1}}{E_{O_2}} = \frac{\cos\theta_1 + n_2/n_1\cos\theta_1}{\cos\theta_1 - n_2/n_1\cos\theta_2} \quad \text{--- (8)}$$

From eq^n (6) for refraction

$$\frac{E_{O_1}}{E_{O_2}} = \frac{\sqrt{\varepsilon_2/\mu_0}\cos\theta_2 + \sqrt{\varepsilon_2/\mu_2}\cos\theta_1}{2\sqrt{\varepsilon_1/\mu_0}\cos\theta_1}$$

$$= \frac{\sqrt{\varepsilon_2/\varepsilon_1}\cos\theta_2 + \cos\theta_1}{2\cos\theta_1}$$

$$\frac{E_{O_1}}{E_{O_2}} = \frac{n_2/n_1\cos\theta_2 + \cos\theta_1}{2\cos\theta_1} \quad \text{--- (9)}$$

using snell's law $\frac{n_2}{n_1} = \frac{\sin\theta_1}{\sin\theta_2}$

$$\frac{E_{O_1}}{E_{O_2}} = \frac{\frac{\sin\theta_1}{\sin\theta_2}\cos\theta_2 + \cos\theta_1}{2\cos\theta_1\sin\theta_2}$$

$$\frac{E_{O_1}}{E_{O_2}} = \frac{\sin(\theta_1+\theta_2)}{2\cos\theta_1\sin\theta_2} \quad \text{—— (10)}$$

For reflection

$$\frac{E_{O_1}}{E_{O_2}} = \frac{\cos\theta_1 + \frac{\sin\theta_1}{\sin\theta_2}\cos\theta_2}{\cos\theta_1 - \frac{\sin\theta_1}{\sin\theta_2}\cos\theta_2}$$

$$\frac{E_{O_1}}{E_{O_2}} = \frac{\sin(\theta_1+\theta_2)}{\sin(\theta_2-\theta_1)} \quad \text{—— (11)}$$

Equation (10) & (11) gives the fresnel's eqn for noncondueting medium

☆ Scattering of E.M wave by free electron :- (Thomson scattering)

When electromagnetic wave is allowed to incident on a system of charge particle. The charge parellel of the oscillates and they absorb incident wave and will be ~~emitted~~ reemitted in the space in all direction. This phenomenon is K/a scattering.

Let us consider an electron of mass 'm' and charge 'e' is kept in a polarished light.

Let the incident electromagnetic wave having intensity along x-dirn and moving along z dirn. The force ef accellerates the electron by accelleration d^2x/dt^2

$$m\frac{d^2x}{dt^2} = eE = eE_0e^{-i(\omega t - kx)} \quad \text{—— (A)}$$