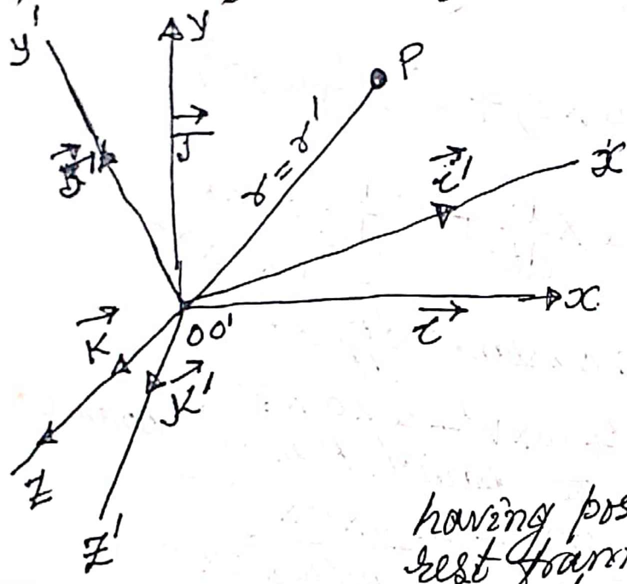
ROTATING FRAME OF REFERANCE

Coriolis's force :- when a body is moving in a rotating frame of referance, a Psedo force is developed which deflects the body perpendicular to the path K/a Corioli's force.

Expression of corioli's force :-



Let us consider two frame of referance in which (x, y, z) is rest frame and $x'y'z'$ is rotating frame. The unit vector of this frame are $(\hat{i}, \hat{j}, \hat{k})$ and $i'j'k'$ respectively.

Let us consider a particle P having position vectors r and r' in rest frame and rotating frame respectively

$$\left.\begin{aligned} r &= \vec{i}x + \vec{j}y + \vec{k}z \\ \& \; r'' &= i'x' + j'y' + k'z' \end{aligned}\right\} \text{—— (A)}$$

Now, $$\left(\frac{dr}{dt}\right)_{fix} = \vec{i}\cdot\frac{dx}{dt} + J\frac{dy}{dt} + k\frac{dz}{dt}$$

Again $$\left(\frac{dr}{dt}\right)_{fix} = \left(\frac{dr'}{dt}\right)_{rot} = i'\frac{dx'}{dt} + J'\frac{dy'}{dt} + k'\frac{dy'}{dt} + x'\frac{di'}{dt} + y'\frac{dJ'}{dt} + z'\frac{dk'}{dt}$$

Now $x'\frac{di'}{dt} = (\omega \times i')x'$, $y'\frac{dJ'}{dt} = (\omega \times J)y'$

$\& \; z'\frac{dk'}{dt} = (\omega \times k')z'$

$$\therefore \left(\frac{dr}{dt}\right)_{fix} = \left(\frac{dx}{dt}\right)_{rot} + x'(\omega \times i') + y'(\omega \times J') + z'(\omega \times k')$$

or, $$\left(\frac{dr}{dt}\right)_{fix} = \left(\frac{dr}{dt}\right)_{rot} + \vec{\omega} \times r' \quad \text{—— (B)}$$

or, $\boxed{V = V' + \omega \times r}$ ——— (C)

The equation is gives a vector operation

$$\left(\frac{d-}{dt}\right)_{fix} = \left(\frac{d-}{dt}\right)_{rot} + \vec{\omega} \times \cdots$$

Putting the velocity V in place of 'r'

$$\left(\frac{dV}{dt}\right)_{fix} = \left(\frac{dV}{dt}\right)_{rot} + \omega \times V$$

$$\text{or, } \left(\frac{dV}{dt}\right)_{fix} = \frac{d}{dt}(V' + \omega \times r) + \omega \times (V' + \omega \times r)$$

$$\text{or, } \left(\frac{dV}{dt}\right)_{fix} = \left(\frac{dV'}{dt}\right)_{rot} + \omega \times \frac{dr}{dt} + r \cdot \frac{d\omega}{dt} + \omega \times V' + \omega \times \omega \times r$$

Now ω is constant $\frac{d\omega}{dt} = 0$

$$\therefore a = a' + \omega \times V' + \omega \times V' + \omega \times \omega \times r$$

The force acting on the particle P in S-frame

$$\boxed{ma' = ma' + 2m\,\omega \times V' + m(\omega \times \omega \times r)} \text{ ——— (D)}$$

$$\text{or, } \boxed{F' = F + 2m(\omega \times V') + m(\omega \times \omega \times r)}$$

The term $m(\omega \times \omega \times r)$ is well known force k/a centrifugal force. The force $2m(\omega \times V')$ is k/a Coriolis's force. The direction of this force is perpendicular to the plane passing through $(\omega \times V')$

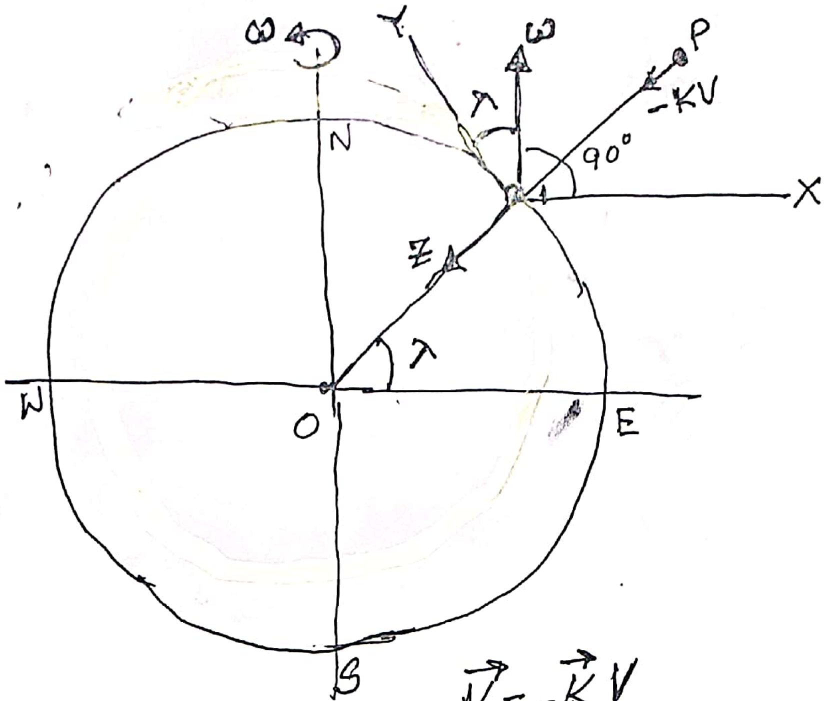
SHIFTING OF FREELY Falling body:-

Let us consider a body of mass m is dropped from the height 'h' at longitude λ. The angular velocity of earth is given by.

$$\omega = \vec{i}\,\omega_x + \vec{J}\,\omega_y + \vec{K}\,\omega_z$$

$$\omega = \vec{i}\,\omega x + \vec{J}\,\omega_y + \vec{K}\,\omega_z.$$

$$\omega = 0 + \vec{J}\,\omega_y + \vec{K}\,\omega_z.$$



The velocity along Z-axis, $\vec{V} = -\vec{K}V$

The coriolis force $F = -2m(\vec{\omega} \times \vec{V})$

$= 2m[(\vec{J}\vec{\omega}\cos\lambda + K\vec{\omega}\sin\lambda)(\vec{K}V)$

$= 2m\omega\cos\lambda V \vec{i}$

Now Putting $V = 0 + gt$

$\therefore F = m \cdot \frac{dx^2}{dt^2} = 2m\omega\cos\lambda\, gt\, \vec{i}$

This shows that body is <u>shifted towards east</u>

$\therefore \frac{d^2x}{dt^2} = 2\omega\cos\lambda\, gt$

on double integrating we get

$x = 2\omega\cos\lambda\, g\frac{t^3}{2\times3}$

$$\boxed{x = \omega g\frac{t^3}{3}\cos\lambda}$$

Again $h = 0 + \frac{1}{2}gt^2 \Rightarrow t = \sqrt{\frac{2h}{g}}$

$\therefore x = \frac{\omega g\cos\lambda}{3}\sqrt{\frac{8h^3}{g^3}}$

$$\boxed{x = \frac{\omega\cos\lambda}{3}\sqrt{\frac{8h^3}{g}}}$$