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SCHRODINGER'S WAVE EQUATION FOR ONE DIMENSIONAL SQUARE POTENTIAL WELL

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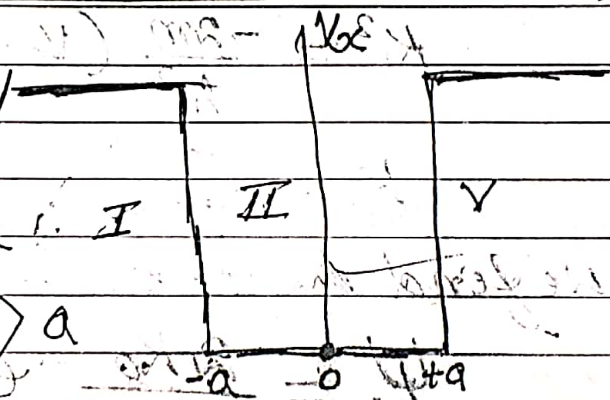
Let a square Potential Well defined as

$V = 0$ for $-a < x < a$ I

$V = V_0$ for $-a > x > a$



The wave equation in the region (I) & region II are given by

$$\frac{d^2\psi_1}{dx^2} + \frac{2m}{\hbar^2} E\psi = 0 \quad \text{——— (I)}$$

and

$$\frac{d^2\psi_2}{dx^2} + \frac{2m}{\hbar^2}(V_0 - E) = 0 \quad \text{——— (ii)}$$

Let $K_0^2 = \sqrt{\frac{2mE}{\hbar^2}}$

& $K^2 = \frac{2m(V_0 - E)}{\hbar^2}$

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(14)

$\therefore \frac{d^2\psi_1}{dx^2} + K_0^2\psi_1 = 0 \Rightarrow \psi_1 = A\cos K_0 x + B\sin K_0 x$ — (3)

& $\frac{d^2\psi_2}{dx^2} - K^2\psi_2 = 0 \Rightarrow \psi_2 = Ce^{Kx} + De^{-Kx}$ — (4)

The boundary condition at $x = \pm\infty$ require that in region $x > a$, $C = 0$ and in the region $x < -a$, $D = 0$

$\therefore \psi_1 = A\cos K_0 x + B\sin K_0 x$ — (5)

$\psi_2 = Ce^{Kx}$ for $x < -a$. — (6)

$\psi_3 = De^{-Kx}$ for $x > a$. — (7)

Where A is amplitude of initial wave, B = amplitude of reflected wave in region I, C = amplitude of penetrating wave in region II
D = amplitude of reflected wave in region II

Applying boundary condition
ψ and $\frac{d\psi}{dt}$ are continuous at $x = \pm a$

i.e $\psi_1 = \psi_3$ & $\frac{d\psi_1}{dt} = \frac{d\psi_3}{dt}$

From eqn 5 & 7

$A\cos K_0 a + B\sin K_0 a = De^{-Ka}$ — (8)

and $-AK_0\sin K_0 a + BK_0\sin K_0 a = -DKe^{-Ka}$ — (9)

Again considering at $x = -a$, $\psi_1 = \psi_2$ & $\frac{d\psi_1}{dt} = \frac{d\psi_2}{dt}$

From eqn (5) & 6, $A\cos K_0 a - B\sin K_0 a = Ce^{-Ka}$ — eqn (10)

and $AK_0\sin K_0 a - BK_0\cos K_0 a = CKe^{-Ka}$ — (11)

Adding and substracting equ^n (8) & (10)

$$2A\cos K_0 a = (C+D)e^{-Ka} \quad \text{—— (12)}$$

and $$2B\sin K_0 a = (D-C)e^{-Ka} \quad \text{—— (13)}$$

Similarly from equ^n (9) & (11)

$$2BK_0\cos K_0 a = (C-D)Ke^{-Ka} \quad \text{—— (14)}$$

and $$2AK_0\sin K_0 a = (C+D)Ke^{-Ka} \quad \text{—— (15)}$$

Deviding (12) by (15)

$$K_0 \tan K_0 a = K \quad \text{—— (16)}$$

Deviding (14) by 13.

$$K_0 \cot K_0 a = -K \quad \text{—— (17)}$$

Let $K_0 a = l$ & $Ka = m$.

From (16) $K_0 a \tan K_0 a = Ka$

$$l \tan l = m \quad \text{—— (18)}$$

$$\therefore l^2+m^2 = K_0^2a^2 + K^2a^2 = a^2(K_0^2+K^2)$$

$$= a^2\left[\frac{2mE}{\hbar^2} + (V-E)\frac{2m}{\hbar^2}\right]$$

$$= \frac{2mVa^2}{\hbar^2} = \text{constant} \quad \text{—— (19)}$$

Now from equ^n (17) $l \cot l = -m$ —— (20)

Thus the energy level depends upon Va^2

In special case when $V \to \infty$ i.e Potential barrier of infinite size.

$$\therefore \tan K_0 a = \frac{K}{K_0} \to \infty$$

or, $K_0 a = (2n+1)\pi/2$, where $n = 1, 2, 3$

or, $\frac{2mE}{\hbar^2}a^2 = (2n+1)^2\frac{\pi^2}{4}$

or, $\boxed{E_n = \frac{(2n+1)^2\pi^2 \hbar^2}{8ma^2}}$ —— (21)

The gives the solution of 1st group. For second group of solution for which $V \to 0$

$$\tan K_0 a = -\frac{K_0}{K} = 0$$

or, $K_0 a = n\pi$

or, $\frac{2mE}{\hbar^2}a^2 = n\pi$

or, $\boxed{E_n = \frac{(2n)^2\pi^2 \hbar^2}{8ma^2}}$ —— (22)

Discus the transmission of a particle through a rectangulal potential barrier. Discus briefly its application to the observed phenomena of α-decay at nuclei. Etc.

कहीं कहीं / Rectangular one dimensional potential barrier.

Let us consider a beam of particle of energy E incident from the left on a potential barrier of height V_0 and width a. The potential energy in region I & III is zero and that in region II is V_0.

Let ψ_1, ψ_2 & ψ_3 are the wave function in region I, II & III respectively.