# Degree:–1(H+S)

## Chapter:– Area of curves length determined from Polar Equation

Topic:– Integral Calculus

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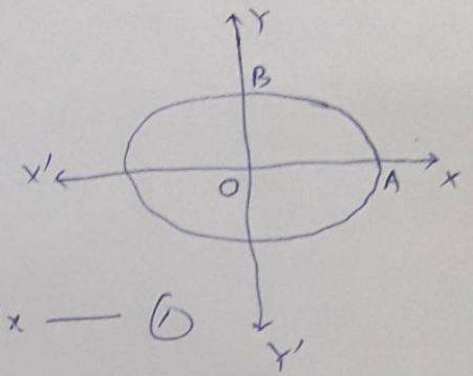
# Area of curves

① Find the area of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$




Soln:- An ellipse is symmetrical about both the axes.

∴ Area of the ellipse = 4 × area of OAB

extent of OAB along x-axis is $O(x=0)$ to $A(x=a)$

∴ Required area of the ellipse $= 4\int_0^a y\,dx$ — (1)

Put, $x = a\cos\theta \Rightarrow dx = -a\sin\theta\,d\theta$

$x = 0 \Rightarrow \cos\theta = 0 = \cos\frac{\pi}{2} \Rightarrow \theta = \pi/2$

$x = a \Rightarrow \cos\theta = 1 = \cos 0^\circ \Rightarrow \theta = 0^\circ$

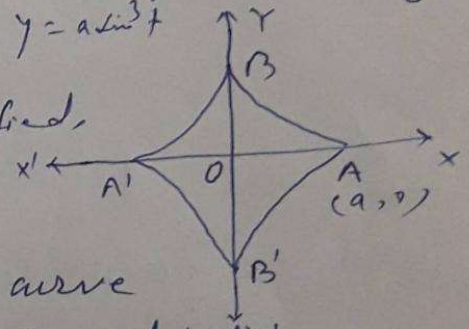
and $y = b\sin\theta$

$$\therefore A = 4\int_0^a b\sin\theta\,(-a\sin\theta\,d\theta) = +4ab\int_0^{\pi/2}\sin^2\theta\,d\theta$$

$$= 4ab\int_0^{\pi/2}\frac{1+\cos 2\theta}{2}\,d\theta = 4ab\times\frac{1}{2}\times\frac{\pi}{2} = \pi ab.$$

② Show that the entire area of the curve $x^{2/3} + y^{2/3} = a^{2/3}$ is $\frac{3}{8}\pi a^2$

OR Find the area of the astroid. $x = a\cos^3 t$, $y = a\sin^3 t$




Soln:- (i) Since by putting $x = 0, y = 0$ both the sides of the curve are not satisfied, hence the curve does not pass through the origin.

(ii) After putting $x = -x$ and $y = -y$, given curve does not change, therefore the curve is symmetrical about both the axes.

(iii) Putting $y = 0 \Rightarrow x^{2/3} = a^{2/3} \Rightarrow x^2 = a^2 \Rightarrow x = \pm a$

i.e. the curve cuts the x-axis at $(a, 0), (-a, 0)$

Similarly $x = 0 \Rightarrow y = \pm a$, the curve cuts the y-axis at $(0, a)$ and $(0, -a)$

Area of the curve = 4 × area of OAB

$$\Rightarrow A = 4\int_0^a y\,dx \quad \text{— (1)}$$

Parametric eqns $x = a\cos^3\theta$ and $y = a\sin^3\theta$

$dx = -3a\cos^2\theta\sin\theta\,d\theta$

Also $x = 0 \Rightarrow \theta = \pi/2$ and $x = a \Rightarrow \theta = 0$

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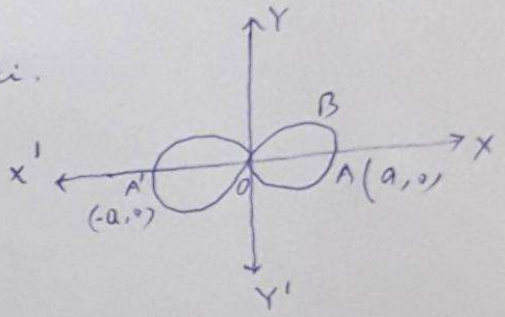
From ① $A = 4\int_{\pi/2}^{0} a\sin^3\theta\,(-3a\sin\theta\cos^2\theta)\,d\theta = 12a^2\int_0^{\pi/2}\sin^4\theta\cos^2\theta\,d\theta$ ②

$A = 12a^2\int_0^{\pi/2}\sin^4\theta\,(1-\sin^2\theta)\,d\theta = 12a^2\int_0^{\pi/2}\sin^4\theta\,d\theta - 12a^2\int_0^{\pi/2}\sin^6\theta\,d\theta$

$= 12a^2\left[\frac{3}{4}\cdot\frac{1}{2}\cdot\frac{\pi}{2} - \frac{5}{6}\cdot\frac{3}{4}\cdot\frac{1}{2}\cdot\frac{\pi}{2}\right] = \frac{3\pi a^2}{8}$ Ans

③ Find the whole area of the curve $a^2y^2 = x^2(a^2-x^2)$.




Sol:- (i) The curve passes through the origin.

(ii) The curve is symmetrical about both the axes.

(iii) It cuts the x-axis at $x = 0, x = \pm a$.
It cuts the x-axis only at $y = 0$.

(iv) There is no portion of the curve beyond $x = a$ or $x = -a$
$y^2$ is (−)ve and $y$ is imaginary.

Area of the whole curve = 4 × area of AB $= 4\int_0^a y\,dx$

$= 4\int_0^a \frac{x}{a}\sqrt{a^2-x^2}\,dx$ —— ①

Put $x = a\sin\theta \Rightarrow dx = a\cos\theta\,d\theta$ Also $x = 0 \Rightarrow \theta = 0$, $x = a \Rightarrow \theta = \pi/2$

① becomes $= 4\int_0^{\pi/2}\sin\theta\cdot a\cos\theta\cdot a\cos\theta\,d\theta = 4a^2\int_0^{\pi/2}\sin\theta\cos^2\theta\,d\theta$ —— ②

Let $\cos\theta = u \Rightarrow -\sin\theta\,d\theta = du$

From ② $= 4a^2\int_1^0 u^2(-du) = 4a^2\int_0^1 u^2\,du = \frac{4a^2}{3}$

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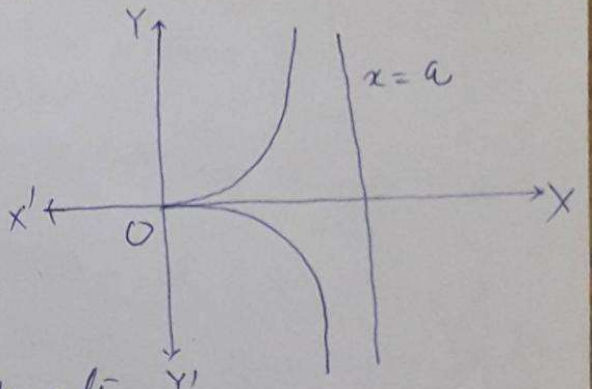
(4) Obtain the area included between the curve $y^2(a-x) = x^3$ and its asymptotes.

Soln:-




(i) The curve passes through the origin, since both the sides are satisfied by putting $x=0, y=0$ in the eqn of the curve.

(ii) The curve is symmetrical about the x-axis.

(iii) The curve does not cut the co-ordinate axes at any other point except at the origin.

(iv) The x-axis is the tangent to the curve at the origin.

(v) The asymptote of the curve parallel to y-axis is $x=a$.

(vi) There is no portion of the curve on the L.H.S of $x=0$ or on the R.H.S of $x=a$.

$\therefore$ Required area = 2 × area of the upper half

$$= 2\int_0^a y\,dx = 2\int_0^a \frac{x\sqrt{x}}{\sqrt{a-x}}\,dx \quad \text{—— (1)}$$

To evaluate (1), put $x = a\sin^2\theta$

$\Rightarrow dx = 2a\sin\theta\cos\theta\,d\theta$

when $x = 0 \Rightarrow \theta = 0$ and $x = a \Rightarrow \theta = \pi/2$.

(1) becomes $= 2\int_0^{\pi/2} \frac{a\sin^2\theta \cdot \sqrt{a}\sin\theta}{\sqrt{a}\cos\theta} \cdot 2a\sin\theta\cos\theta\,d\theta$

$$= 4a^2\int_0^{\pi/2} \sin^4\theta\,d\theta = 4a^2 \cdot \frac{3}{4}\cdot\frac{1}{2}\cdot\frac{\pi}{2} = \frac{3\pi a^2}{4} \text{ Ans}$$

(5) Find the area of the cycloid $x = a(\theta - \sin\theta)$, $y = a(1-\cos\theta)$ from cusp to cusp.

Soln:- From the curve $y=0$ when $\theta = 0$ to $2\pi$.

$x = a(\theta - \sin\theta) \Rightarrow dx = a(1-\cos\theta)\,d\theta$

Required area $= \int y\,dx = \int_0^{2\pi} a(1-\cos\theta)\cdot a(1-\cos\theta)\,d\theta = a^2\int_0^{2\pi}(1-\cos\theta)^2\,d\theta$

$$= a^2\int_0^{2\pi}(1+\cos^2\theta - 2\cos\theta)\,d\theta = a^2\int_0^{2\pi}\left(1 + \frac{1+\cos 2\theta}{2} - 2\cos\theta\right)d\theta$$

$$= \frac{a^2}{2}\int_0^{2\pi}(3 - 4\cos\theta + \cos 2\theta)\,d\theta = 3\pi a^2 \text{ sq}$$

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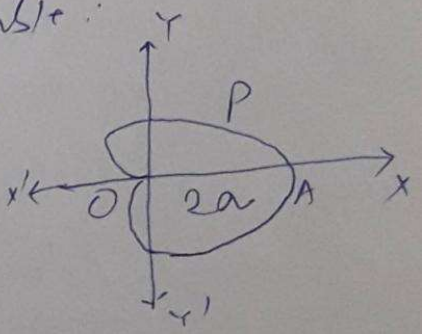
(6) Find the area of the cardiode $r = a(1+\cos\theta)$

Sol:- (i) Replacing $-\theta$ for $\theta$ in the eqn of the given curve, the eqn does not change. Therefore the curve is symmetrical about the initial line.

(ii) $r = 0 \Rightarrow \theta = \pi$ ; $\theta = 0 \Rightarrow r = 2a$.

As $\theta$ increases from 0 to $\pi$, $r$ decreases from $2a$ to $a$ as is evident from the corresponding values of $r$ and $\theta$ given by the following table:

| $\theta$ | 0 | $30^\circ$ | $45^\circ$ | $60^\circ$ | $90^\circ$ | ... | $180^\circ$ |
|---|---|---|---|---|---|---|---|
| $r$ | $2a$ | $a\left(1+\frac{\sqrt{3}}{2}\right)$ | $a\left(1+\frac{1}{\sqrt{2}}\right)$ | $\frac{3a}{2}$ | $a$ | ... | 0 |




$\therefore$ Required area = 2 × area of $AP = 2 \times \frac{1}{2}\int_0^\pi r^2 d\theta$

$$= \int_0^\pi a^2(1+\cos\theta)^2 d\theta = a^2\int_0^\pi (1+2\cos\theta+\cos^2\theta)\,d\theta$$

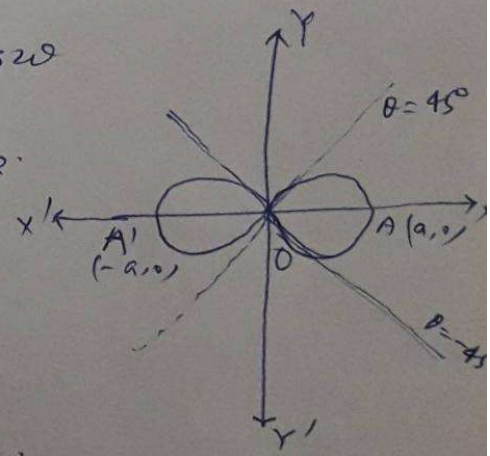
$$= a^2\left[\pi + 2\{\sin\theta\}_0^\pi + 2\int_0^{\pi/2}\cos^2\theta\, d\theta\right]$$

$$= a^2\left[\pi + 0 + 2\times\frac{1}{2}\times\frac{\pi}{2}\right] = \frac{3}{2}\pi a^2$$

(7) Find the area of the loop of the curve $r^2 = a^2\cos 2\theta$

Sol:- (i) Replacing $-\theta$ for $\theta$, curve does not change i.e. curve is symmetrical about the initial line.

(ii) $r = 0 \Rightarrow \theta = \pm\pi/4$ and $\theta = 0 \Rightarrow r = \pm a$.




$\theta = 30^\circ$, $r = \pm a/\sqrt{2}$ , $\theta = 45^\circ$, $r = 0$

$\theta = 60^\circ$, $r^2 = -ve$. , $\theta = 90^\circ$, $r^2 = -ve$.

Put $r\cos\theta = x$ and $r\sin\theta = y$ in $r^2 = a^2\cos 2\theta$.

$\Rightarrow r^2 = a^2(\cos^2\theta - \sin^2\theta) \Rightarrow (x^2+y^2)^2 = a^2(x^2-y^2)$

Equating the lowest degree term to zero we get $x^2 - y^2 = 0 \Rightarrow y = \pm x$.

therefore the tangents to the curve at the origin are $y = \pm x$.

Area of one loop = 2 × area $OAP = 2 \times \frac{1}{2}\int_0^{\pi/4} r^2 d\theta = \int_0^{\pi/4} a^2\cos 2\theta\, d\theta$

$$= a^2\left[\frac{\sin 2\theta}{2}\right]_0^{\pi/4} = \frac{a^2}{2}.$$

Required area = area of the two equal loops

$$= 2 \times \frac{a^2}{2} = a^2 \quad \text{sq units}$$

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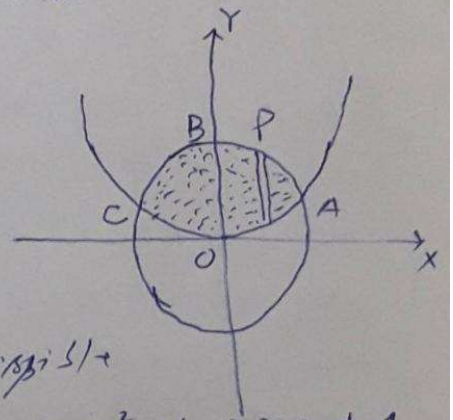
(8) (8) Find the area common to the circle $x^2+y^2=25$ and the parabola $3x^2=16y$. (5)

Sol:- $x^2+y^2=25$ represents a circle with centre $(0,0)$, radius $= 5$

$\boxed{x^2+y^2=a^2}$

and $3x^2=16y$ represents a parabola $\boxed{x^2=4ay}$ whose vertex is the origin and the axis is the y-axis.




Solving $x^2+y^2=25$ and $3x^2=16y$ we get the points of intersection of the two curves.

$\frac{16}{3}y+y^2=25 \Rightarrow 3y^2+16y-75=0$

$\Rightarrow (y-3)(3y+25)=0$

$\therefore y=3, -\frac{25}{3}$ (-)ve value is inadmissible

$\therefore y=3$ and $x^2+y^2=25 \Rightarrow x^2+9=25 \Rightarrow x^2=16 \Rightarrow x=\pm 4$

Thus the co-ordinates of $A=(4,3)$.

Now the area common to the two curves $=$ 2 x area AOB

$$= 2\int_0^4 (y_1-y_2)\,dx$$

where $y_1$ is taken for the circle $x^2+y^2=25$ and $y_2$ is taken for the parabola $3x^2=16y$

$$= 2\times\int_0^4 \left(\sqrt{25-x^2}-\frac{3}{16}x^2\right)dx$$

$$= 2\left[\left\{\frac{x\sqrt{25-x^2}}{2}+\frac{25}{2}\sin^{-1}\frac{x}{5}\right\}-\frac{3}{16}\cdot\frac{x^3}{3}\right]_0^4$$

$$= 2\left[6+\frac{25}{2}\sin^{-1}\left(\frac{4}{5}\right)-4\right] = 4+25\sin^{-1}(4/5)$$ sq

(9) Find the area enclosed by the parabola $y^2=4ax$ and $x^2=4by$.

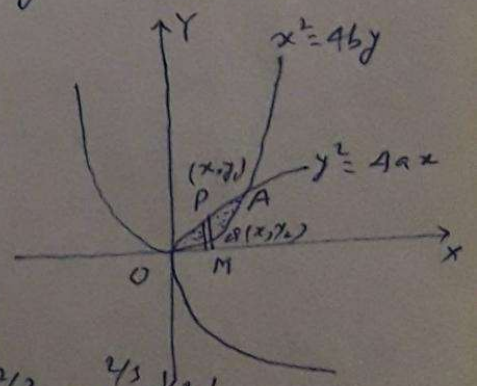
Sol:- $y^2=4ax$ — (1) and $x^2=4by$ — (2)

$\Rightarrow y^4=16a^2x^2$

$=16a^2\cdot 4by$ {from (2)}

$\Rightarrow y(y^3-64a^2b)=0 \Rightarrow y=0$ & $4a^{2/3}b^{1/3}$

Now, $y=0 \Rightarrow x=0$ and from (2) $y=4a^{2/3}b^{1/3} \Rightarrow x=4a^{1/3}b^{2/3}$




$\therefore$ Points of intersection are $O(0,0)$ and $A(4a^{1/3}b^{2/3}, 4a^{2/3}b^{1/3})$

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Therefore the area enclosed between the two parabola is

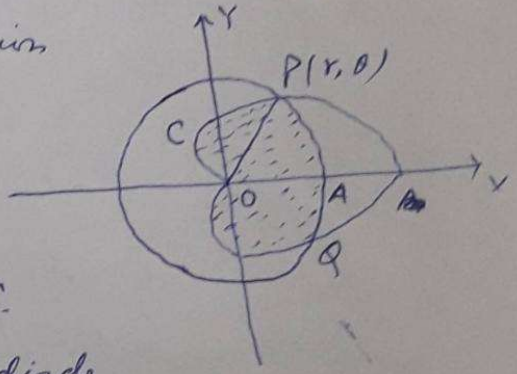
$$= \int_0^{4a^{1/3}b^{2/3}} (y_1 - y_2)\,dx = \int_0^{4a^{1/3}b^{2/3}} \left(\sqrt{4ax} - \frac{x^2}{4b}\right) dx$$

Since $y_1$ lies on $y^2 = 4ax$ and $y_2$ lies on $x^2 = 4by$

$$= \left[2\sqrt{a}\cdot\frac{2}{3}x^{3/2} - \frac{1}{12}x^3\right]_0^{4a^{1/3}b^{2/3}} = \left[\frac{32}{3}ab - \frac{16}{3}ab\right] = \frac{16}{3}ab$$

(12) Find the area common to the circle $r = \frac{3}{2}a$ and the cardiode $r = a(1+\cos\theta)$ and also the area of the remainder of the cardiode.

Soln:- Let $P(r,\theta)$ be the point of intersection of the circle and the cardiode.




Now $r = a(1+\cos\theta)$ and $r = \frac{3}{2}a$

$$\Rightarrow \frac{3}{2}a = a(1+\cos\theta) \Rightarrow 1+\cos\theta = \frac{3}{2}$$

$$\cos\theta = \frac{3}{2} - 1 = \frac{1}{2} = \cos 60^\circ \Rightarrow \theta = 60^\circ$$

The area common to the circle and the cardiode

$$= 2\times \text{area } OAPCO = 2(\text{area } OAP + \text{area } OPCO)$$

$$\text{Area } OAP = \frac{1}{2}\int_0^{\pi/3} r^2\,d\theta \quad \text{where } r = \frac{3}{2}a$$

$$= \frac{1}{2}\int_0^{\pi/3} \frac{9}{4}a^2\,d\theta = \frac{9a^2}{8}\int_0^{\pi/3} d\theta = \frac{9a^2}{8}\cdot\frac{\pi}{3} = \frac{3\pi a^2}{8}$$

$$\text{Area } OPCO = \frac{1}{2}\int_{\pi/3}^{\pi} r^2\,d\theta, \quad \text{where } r = a(1+\cos\theta)$$

$$= \frac{1}{2}\int_{\pi/3}^{\pi} a^2(1+\cos\theta)^2\,d\theta = \frac{a^2}{2}\int_{\pi/3}^{\pi}(1+2\cos\theta+\cos^2\theta)\,d\theta = \frac{a^2}{2}\int_{\pi/3}^{\pi}\left(1+2\cos\theta+\frac{1+\cos 2\theta}{2}\right)d\theta$$

$$= \frac{a^2}{4}\int_{\pi/3}^{\pi}(2+4\cos\theta+1+\cos 2\theta)\,d\theta = \frac{a^2}{4}\left[3\theta + 4\sin\theta + \frac{\sin 2\theta}{2}\right]_{\pi/3}^{\pi}$$

$$= \frac{a^2}{4}\left[\left\{3\pi + 4\sin\pi + \frac{1}{2}\sin 2\pi\right\} - \left\{3\left(\frac{\pi}{3}\right) + 4\sin\frac{\pi}{3} + \frac{1}{2}\sin\frac{2\pi}{3}\right\}\right]$$

$$= \frac{a^2}{4}\left(2\pi - \frac{9\sqrt{3}}{4}\right)$$

$$\therefore \text{Required area} = 2\left\{\frac{3\pi a^2}{8} + \frac{a^2}{4}\left(2\pi - \frac{9\sqrt{3}}{4}\right)\right\} = \left(\frac{7\pi}{4} - \frac{9\sqrt{3}}{8}\right)a^2$$

Area of the remainder cardiode = (whole area of card. − area common to circle & cardiode) $= \frac{3}{2}\pi a^2 - \left(\frac{7\pi}{4} - \frac{9\sqrt{3}}{8}\right)a^2 = \left(\frac{9\sqrt{3}}{8} - \frac{\pi}{4}\right)a^2$

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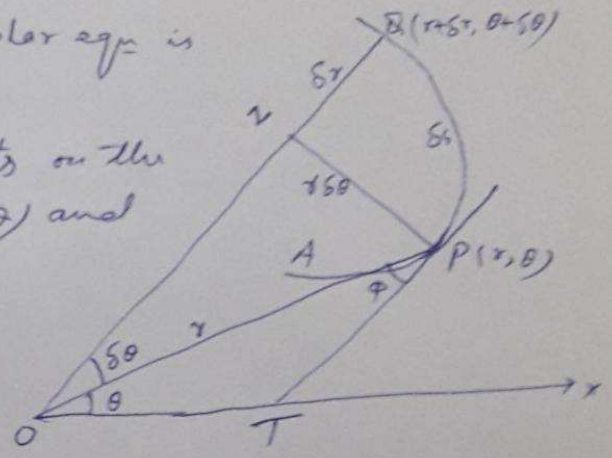
# Lengths Determined from Polar Equation

Art Let AB be a curve whose polar eqn is

$$r = f(\theta)$$




Let P and Q be two consecutive points on the curve whose co-ordinates are $(r, \theta)$ and $(r+\delta r, \theta+\delta\theta)$ respectively so that

$\angle POQ = \delta\theta$ and $OQ = r + \delta r$

Let $AP = s$, then

$AQ = s + \delta s \Rightarrow PQ = \delta s$

Draw $PN \perp OQ$. Then $PN = r\sin\delta\theta$ and $ON = r\cos\delta\theta$.

In the limit when $Q \to P$, $\delta\theta \to 0$ so that PN becomes $= r\delta\theta$ and $ON = r$, consequently $NQ = \delta r$.

Also $\dfrac{\text{arc } PQ}{\text{chord } PQ} \to 1$

In rt. angled $\triangle PNQ$,

$$PQ^2 = PN^2 + NQ^2$$

$$(\delta s)^2 = (r\delta\theta)^2 + (\delta r)^2$$

$$\Rightarrow \left(\frac{\delta s}{\delta\theta}\right)^2 = r^2 + \left(\frac{\delta r}{\delta\theta}\right)^2$$

Taking limit $\delta\theta \to 0$ $\therefore \left(\dfrac{ds}{d\theta}\right)^2 = r^2 + \left(\dfrac{dr}{d\theta}\right)^2$

$$\therefore \frac{ds}{d\theta} = \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2} \Rightarrow \int ds = \int \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\, d\theta.$$

$$\therefore \boxed{s = \int_A^B \sqrt{r^2 + (dr/d\theta)^2}\, d\theta}$$

Let the angle between the radius vector OP and the tangent PT at P be $\phi$.

Then from differential Calculus, $\tan\phi = r\dfrac{d\theta}{dr}$.

$$\sin\phi = \frac{r\,d\theta}{ds}, \quad \cos\phi = \frac{dr}{ds}.$$

Again, $(\delta s)^2 = (\delta r)^2 + (r\delta\theta)^2$

As $\lim \delta r \to 0$ $\left(\dfrac{ds}{dr}\right)^2 = 1 + r^2\left(\dfrac{d\theta}{dr}\right)^2$

$$\Rightarrow \int ds = \int \sqrt{1 + r^2\left(\frac{d\theta}{dr}\right)^2}\, dr$$

$$\therefore \boxed{s = \int_{r_1}^{r_2} \sqrt{1 + r^2\left(\frac{d\theta}{dr}\right)^2}\, dr.}$$

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(Art) Lengths Determined from Pedal Equation 

Let the Pedal eqn of the curve be $p = f(r)$

Then from diff. Calculus, $\frac{dr}{ds} = \cos\phi$, $r\frac{d\theta}{dr} = \tan\phi$

and ~~where~~ $p = r\sin\phi$ where $p$ = length of the $\perp^r$ from the pole to the tangent.

Now $(ds)^2 = (dr)^2 + (r\,d\theta)^2$

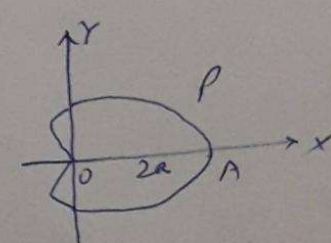
$$\Rightarrow \left(\frac{ds}{dr}\right)^2 = 1 + \left(r\frac{d\theta}{dr}\right)^2 = 1 + \tan^2\phi = \sec^2\phi = \frac{1}{\cos^2\phi}$$

$$= \frac{1}{1-\sin^2\phi} = \frac{1}{1-\frac{p^2}{r^2}} = \frac{r^2}{r^2-p^2}.$$

$$\frac{ds}{dr} = \frac{r}{\sqrt{r^2-p^2}} \Rightarrow \int ds = \int \frac{r\,dr}{\sqrt{r^2-p^2}}$$

$$\therefore \boxed{s = \int_{r_1}^{r_2} \frac{r\,dr}{\sqrt{r^2-p^2}}}$$

Ex-1 Find the entire length of the Cardiode $r = a(1+\cos\theta)$.




Soln:- $r = a(1+\cos\theta)$

$$\frac{dr}{d\theta} = -a\sin\theta$$

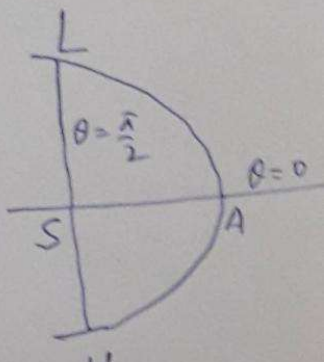
$$\therefore \text{whole length} = 2\int_0^{\pi} \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\, d\theta$$

$$= 2\int_0^{\pi} \sqrt{a^2(1+\cos\theta)^2 + a^2\sin^2\theta}\, d\theta = 2a\int_0^{\pi} \sqrt{2+2\cos\theta}\, d\theta$$

$$= 2a\int_0^{\pi} \sqrt{2\times 2\cos^2\frac{\theta}{2}}\, d\theta = 4a\int_0^{\pi} \cos\frac{\theta}{2}\, d\theta = 4a\cdot 2\left[\sin\frac{\theta}{2}\right]_0^{\pi}$$

$$= 8a$$

**Ex-2** Find the length of the parabola $\frac{2a}{r} = 1 + \cos\theta$ cut off by the latus rectum.




Soln:- Focus S is the pole

$$\therefore \frac{2a}{r} = 1 + \cos\theta$$

$$r = \frac{2a}{1+\cos\theta} = \frac{2a}{2\cos^2\theta/2} = a\sec^2\frac{\theta}{2}.$$

So that $\frac{dr}{d\theta} = a \cdot 2\sec\frac{\theta}{2} \cdot \sec\frac{\theta}{2} \cdot \tan\frac{\theta}{2} \cdot \frac{1}{2}$

$$= a\sec^2\frac{\theta}{2} \cdot \tan\frac{\theta}{2}$$

The length of the arc $LAL' = 2 \times \text{arc } AL = 2\int_0^{\pi/2} \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\, d\theta$

$$= 2\int_0^{\pi/2} \sqrt{a^2\sec^4\frac{\theta}{2} + a^2\sec^4\frac{\theta}{2}\tan^2\frac{\theta}{2}}\, d\theta$$

$$= 2\int_0^{\pi/2} \sqrt{a^2\sec^4\frac{\theta}{2}\left(1+\tan^2\frac{\theta}{2}\right)}\, d\theta = 2a\int_0^{\pi/2} \sec^2\frac{\theta}{2}\sqrt{1+\tan^2\frac{\theta}{2}}\, d\theta \quad \text{---(1)}$$

Let $\tan\frac{\theta}{2} = u$ so that $\sec^2\frac{\theta}{2} \cdot \frac{d\theta}{2} = du$

Also $\theta = 0 \Rightarrow u = 0$ and $\theta = \frac{\pi}{2} \Rightarrow u = \tan\frac{\pi}{4} = 1$

$\therefore$ The required length of arc, from (1) $= 2a\int_0^1 \sqrt{1+u^2} \cdot 2\,du$

$$= 4a\left[\frac{u}{2}\sqrt{1+u^2} + \frac{1}{2}\log\left\{u + \sqrt{1+u^2}\right\}\right]_0^1$$

$$= 4a\left[\frac{1}{2}\sqrt{2} + \frac{1}{2}\log(1+\sqrt{2})\right] = 2a\left\{\sqrt{2} + \log(1+\sqrt{2})\right\}.$$

**Ex-3** Find the intrinsic eqn of the curve $r = a(1-\cos\theta)$

Soln:- $r = a(1-\cos\theta)$

$$\frac{dr}{d\theta} = a\sin\theta$$

$$\frac{1}{r}\cdot\frac{dr}{d\theta} = \frac{a\sin\theta}{a(1-\cos\theta)} = \frac{2\sin\theta/2\cos\theta/2}{2\sin^2\theta/2}$$

$$\frac{1}{\tan\phi} = \cot\frac{\theta}{2} \Rightarrow \cot\phi = \cot\frac{\theta}{2}$$

$$\therefore \phi = \frac{\theta}{2}$$

**Working rule**

(i) $s = \int_\alpha^\theta \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\, d\theta$

(ii) $\psi = \theta + \phi$

(iii) $\tan\phi = r\frac{d\theta}{dr} = \frac{r}{dr/d\theta}$

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Consequently $\psi = \theta + \phi = \theta + \frac{3\theta}{2} = \frac{3\theta}{2}$ — (1) (10)

Also $s = \int_0^\theta \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\, d\theta$

$= \int_0^\theta \sqrt{a^2(1-\cos\theta)^2 + a^2\sin^2\theta}\, d\theta = a\int_0^\theta \sqrt{2(1-\cos\theta)}\, d\theta$

$= \sqrt{2}\,a \int_0^\theta \sqrt{2\sin^2\frac{\theta}{2}}\, d\theta = 2a\int_0^\theta \sin\frac{\theta}{2}\, d\theta = \cancel{2a\left[-2\cos\frac{\theta}{2}\right]_0^\theta}$

$= 2a\left[-2\cos\frac{\theta}{2}\right]_0^\theta = 4a\left(1-\cos\frac{\theta}{2}\right)$ — (2)

Eliminating $\theta$ between (1) & (2) we get

$s = 4a\left(1-\cos\frac{\psi}{3}\right)$ which is the intrinsic eqn of the curve.

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