

SPIN HALF ANGULAR MOMENTUM

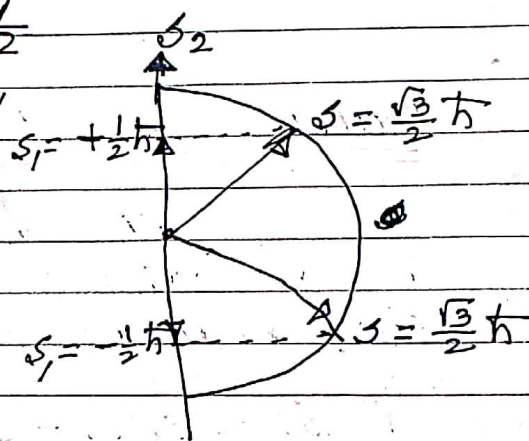
The electron spins about an axis passing through its centre of mass. Therefore it has an internal or intrinsic angular momentum and a magnetic moment associated with spin rotation. The spin angular momentum of electron is given by

$$S = \sqrt{s(s+1)} \hbar = \sqrt{\frac{1}{2}(\frac{1}{2}+1)} \hbar$$

where s is spin quantum number and l is orbital quantum number. It has only one value i.e. $\frac{1}{2}$

In the presence of external magnetic field, the component of the angular momentum is given by

$$S_z = m_s \hbar = \pm \frac{1}{2} \hbar$$



where m_s is called spin magnetic quantum number.

The magnetic moment associated with the spin angular momentum is given by

$$\vec{\mu}_s = 2s \frac{e}{2m} \sqrt{s(s+1)} \hbar = \mu_B \sqrt{s(s+1)}$$

where $\mu_B = \text{Bohr Magnetron} = \frac{e\hbar^2}{2m}$

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The z-component of magnetic moment

$$\begin{aligned} (\mu_s)_z &= 2 \frac{e}{2m} \hbar m_2 = 2 \frac{e}{2m} \hbar m_2 \\ &= \pm \frac{e}{2m} \hbar \left[\because m_2 = \pm \frac{1}{2} \right] \end{aligned}$$

Thus $(\mu_s)_z$ has only two values $\frac{e}{2m} \hbar$ and $-\frac{e}{2m} \hbar$ has been verified by Stern-Gerlach experiment.

Pauli's Spin Matrices

The spin operators S_x, S_y, S_z are associated with the component of spin angular momentum as

$$\left. \begin{aligned} [S_x, S_y] &= S_x S_y - S_y S_x = i \hbar S_z \\ [S_y, S_z] &= (S_y S_z - S_z S_y) = i \hbar S_x \\ [S_z, S_x] &= (S_z S_x - S_x S_z) = i \hbar S_y \end{aligned} \right\} \text{--- (1)}$$

If we consider the case of spin for electron each of operators must have just two eigen values $\frac{1}{2} \hbar$ & $-\frac{1}{2} \hbar$

$$i.e. S_x = \frac{1}{2} \hbar \sigma_x, S_y = \frac{1}{2} \hbar \sigma_y \text{ \& } S_z = \frac{1}{2} \hbar \sigma_z \text{--- (2)}$$

Where $\sigma_x, \sigma_y, \sigma_z$ are arbitrary operators

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The operators $\sigma_x^2, \sigma_y^2, \sigma_z^2$ must have only the eigen value ± 1 . i.e these are unit operators
 $\sigma_x^2 = \sigma_y^2 = \sigma_z^2 = 1$

The commutation rules are

$$\left. \begin{aligned} [\sigma_x, \sigma_y] &= [\sigma_x \sigma_y - \sigma_y \sigma_x] = 2i\hbar \sigma_z \\ [\sigma_y, \sigma_z] &= [\sigma_y \sigma_z - \sigma_z \sigma_y] = 2i\hbar \sigma_x \\ [\sigma_z, \sigma_x] &= [\sigma_z \sigma_x - \sigma_x \sigma_z] = 2i\hbar \sigma_y \end{aligned} \right\} \text{(3)}$$

$$\begin{aligned} \text{Now } 2i\hbar (\sigma_x \sigma_y - \sigma_y \sigma_x) &= 2i\hbar \cdot \sigma_x \sigma_y + \sigma_y \cdot 2i\hbar \sigma_x \\ &= (\sigma_y \sigma_z - \sigma_z \sigma_y) \sigma_y \\ &\quad + \sigma_y [\sigma_y \sigma_z - \sigma_z \sigma_y] \\ &= 0 \end{aligned}$$

$$\Rightarrow \sigma_x \sigma_y + \sigma_y \sigma_x = 0$$

Similarly $\sigma_y \sigma_z + \sigma_z \sigma_y = 0$

$$\sigma_z \sigma_x + \sigma_x \sigma_z = 0$$

From equⁿ (3) & (4) we get

$$\left. \begin{aligned} \sigma_x \sigma_y &= i \sigma_z \\ \sigma_y \sigma_z &= i \sigma_x \\ \sigma_z \sigma_x &= i \sigma_y \end{aligned} \right\} \text{(5)}$$

Since each σ have two ~~value~~ eigen values
 so 2×2 matrix having the eigen value ± 1

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$$\sigma_z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$\sigma_x = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$\sigma_y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$$

Here $\sigma_x, \sigma_y, \sigma_z$ are called Pauli's spin matrices associated with the component of spin angular momentum

$$S_x = \frac{1}{2} \hbar \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$S_y = \frac{1}{2} \hbar \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$$

$$S_z = \frac{1}{2} \hbar \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$