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(a) Electromagnetic wave :- The light wave is combination of electric field vector (E) and magnetic field vector (H). They are $\perp r$ to each other.

If the component of E along x, y and z axis are E_x, E_y & E_z.

$$\Delta E = \text{div}.E = \frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z}$$

and

$$\text{Curl}.E = i\left(\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z}\right) + j\left(\frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x}\right) + k\left(\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y}\right)$$

(b) Current density :- The electric current per unit area is known as current density (J).

$$J = I/A \Rightarrow I = JA$$

for variable current passing through the area ds.

$$I = \int \vec{J} \cdot \vec{ds}$$

(c) Equation of Continuity :- The average current passing through any area ds.

$$I = \int \vec{J} \cdot \vec{ds} = \int \frac{dq_{in}}{dt}$$

where dq_{in} = charge inside the surface

if ρ = charge density

then $q_{in} = \int_v \rho . dv$

Now, $I = \int_s \vec{J} \cdot \vec{ds} = \frac{d}{dt} \int_v \rho dv$ —— (A)

Using Gauss divergance theorem

$$\oint_s \vec{F} \cdot \vec{ds} = \int_{v} (div. \vec{F}) . dv$$

$$\therefore \int_s \vec{J} \cdot ds = \int_v (div \vec{J}) dv = \int_v (\nabla \cdot J) dv$$

from eq^n (A)

$$I = \int (\nabla \cdot J) dv = \frac{d}{dt} \int_v \rho dv$$

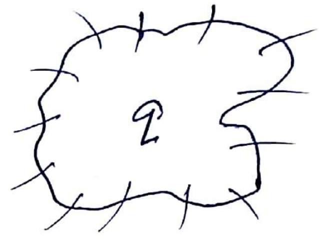
$$(\nabla \cdot J) = \frac{d\rho}{dt}$$

$$\nabla \cdot J - \frac{d\rho}{dt} = 0$$

This is known as equation of continuity

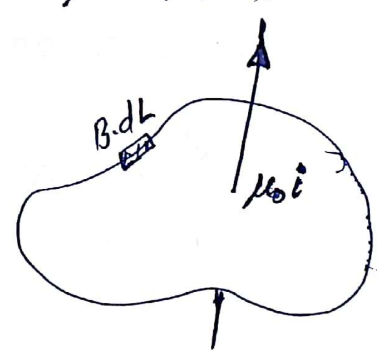
(d) Gauss's Theorem:- The total electric field flux over the closed surface is equal to the $\frac{1}{\varepsilon_0}$ times the charge on the closed surface.

$$\phi = \frac{q}{\varepsilon_0}$$



(e) Ampere's Law:- The line integral of magnetic induction over a closed circuit is equal to μ_0 times the current threed.

$$\oint B \cdot dL = \mu_0 i$$



(f) Electrical displacement:- If an electric field is applied any conductor, there is a displacement of electron & proton.

This displacement is given by

$$\vec{D} = \varepsilon_0 \vec{E}$$

Maxwell's Electromagnetic Equation

There ar four E.M.E -

(i) Law of displacement:- According to Gauss's theorem the electric flux passing through the closed surface is equal to the $\frac{1}{\varepsilon_0}$ times the charge.

$$\phi = \frac{q}{\varepsilon_0}$$

$$\oint \vec{E}\cdot\vec{ds} = \frac{q}{\varepsilon_0}$$

If ρ = charge density then $q = \int_V \rho\, dv$

$$\therefore \oint \vec{E}\vec{ds} = \frac{1}{\varepsilon_0}\int_V \rho\, dv \quad \text{———} \quad (i)$$

Now $\phi = \vec{i}.\frac{\partial \vec{E}}{\partial x} + \vec{j}\frac{\partial \vec{E}}{\partial y} + \vec{k}\frac{\partial \vec{E}}{\partial z}$

The total electric flux passing, the small volume $dx.dy.dz$ is given by-

$$\phi = \vec{i}.\frac{\partial E}{\partial x} + \vec{j}\frac{\partial E}{\partial y} + \vec{k}\frac{\partial E}{\partial z}$$

$$= \left(\vec{i}\frac{\partial}{\partial x} + \vec{j}\frac{\partial}{\partial y} + \vec{k}\frac{\partial}{\partial z}\right)E$$

$$= \vec{\nabla}.\vec{E}$$

$$= \text{div } E$$

Now from $\phi = \text{div } E = \frac{q}{\varepsilon_0} = \frac{1}{\varepsilon_0} \int_v \rho \, dv$

$\because$ Electric displacement, $D = \varepsilon_0 E$ & for

dv = unit volume

$\therefore \text{div}(\varepsilon_0 E) = \rho$

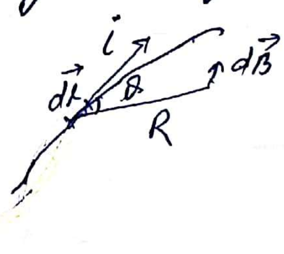
$\text{div} D = \rho$

where ρ = charge density

(ii) Law of magnetic inductance:-

According to Laplace's law the magnetic induction at a point due to current element is given by

$$dB = \frac{\mu_0}{4\pi} \cdot \frac{i \, dl \sin\theta}{R^2}$$



for closed circuit total magnetic induction

$$B = \frac{\mu_0 i}{4\pi} \int \frac{1}{R^2} \cdot \vec{dl} \, \hat{R} \sin\theta$$

$$= \frac{\mu_0}{4\pi} \int \frac{(JA)}{R^2} \cdot dl \sin\theta \cdot \hat{R}$$

$$= \frac{\mu_0}{4\pi} \int \frac{J \cdot dv}{R^2} \sin\theta \cdot \hat{R}$$

$$= \frac{\mu_0}{4\pi} \int \frac{\vec{J} \times \hat{R}}{R^2} \cdot dv$$

Now taking the divergant of B

$$\vec{\nabla} \cdot \vec{B} = \frac{\mu_0}{4\pi} \int \frac{\nabla \cdot (J \times R) \, dv}{R^2}$$

If J is constant then $\nabla \cdot J = 0$

$$\Rightarrow \vec{\nabla} \cdot \vec{B} = 0$$

(iii) Law of electric field:- The magnetic flux linked with area ds is given by-

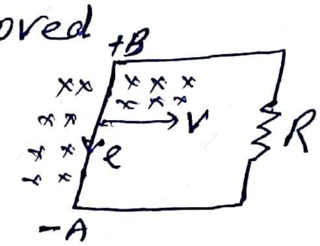
$$\phi = \int_S \vec{B} \cdot \vec{ds}$$

Induced emf,

$$e = -\frac{d\phi}{dt} = -\frac{d}{dt}\left[\int_S \vec{B} \cdot \vec{ds}\right]$$

$$e = -\int \frac{d\vec{B}}{dt} \cdot \vec{ds} \quad \text{——— (A)}$$

If a rod of length is L moved with velocity v in a $\perp r$ magnetic field B then



$$e = -BLV$$

if E = electric intensity inside the rod to move the electron then work done

$$W = \int_A^B E\, dL$$

This work done is equivalent to induced e.m.f.

$$e = \oint_w \vec{E} \cdot \vec{dL} = \int_S (\nabla \times E)\, \vec{ds} \quad \text{——— (B)}$$

comparing eq^n (A) and (B)

$$\boxed{\nabla \times E = -\frac{dB}{dt}}$$

(iv) Law of Magnetic field:-

According to Ampere's law, line integral of magnetic induction is equal to μ_0 times the current thread

$$\oint \vec{B} \cdot \vec{dl} = \mu_0 I$$

$$\oint \vec{H} \cdot \vec{dl} = I \quad \text{——— (1)}$$

Now current density

$$J = \epsilon_0 \cdot \frac{dE}{dt} = \frac{d}{dt}(\epsilon_0 E)$$

$$= \frac{dD}{dt}$$

Now current $I = \int_S \vec{J} \cdot \vec{ds}$

from (1) $\oint \vec{H} \cdot \vec{dl} = \int_S J \cdot ds$ + current density to dielectric polarisation.

$$= \int_S J \cdot ds + \int \frac{dD}{dt} \cdot ds$$

Using Stock's Theorem

$$\oint_C H \cdot dl = \oint_S (\nabla \times H) \vec{ds}$$

$$\Rightarrow \oint_S (\nabla \times H) \vec{ds} = \int_S J \cdot ds + \int \frac{dD}{dt} \cdot ds$$

$$\nabla \times H = J + \frac{dD}{dt}$$

(v) Pointing vector:- The cross product of E & H is known as pointing vector.

We know that

$$\vec{\nabla}(\vec{E}\times\vec{H}) = \vec{E}(\nabla\times H) + \vec{H}(\nabla\times E)$$

$$\because \nabla\times H = J + \frac{dD}{dt}$$

$$\& \ \nabla\times E = -\frac{dB}{dt}$$

$$\therefore \nabla[E\times H] = -E\left(J + \frac{\partial D}{\partial t}\right) + H\cdot\frac{\partial B}{\partial t}$$

$$= -\vec{E}\cdot\vec{J} - \left(E\cdot\frac{\partial D}{\partial t} + H\cdot\frac{\partial B}{\partial t}\right)$$

for free space $J = 0$

$$\therefore \nabla(E\times H) = -\left[E\frac{\partial D}{\partial t} + H\frac{\partial B}{\partial t}\right]$$

$$= -\left[E\frac{\partial(\varepsilon_0 E)}{\partial t} + H\frac{\partial(\mu_0 H)}{\partial t}\right]$$

$$= -\frac{1}{2}\cdot\frac{d}{dt}\left[\varepsilon_0 E^2 + \mu_0 H^2\right]$$

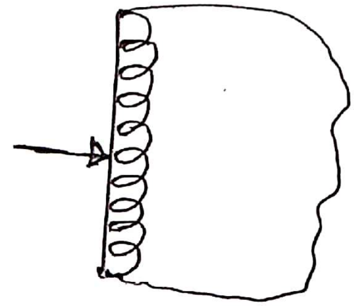
Integrating this vector for volume 'v' bounded by the surface 's'.

$$\oint_s \nabla\cdot(E\times H)ds = -\frac{1}{2}\cdot\frac{d}{dt}\oint_v \frac{1}{2}(\varepsilon_0 E^2 + \mu_0 H^2)dv$$

$$\oint_s \nabla\cdot P ds = -\frac{1}{2}\frac{d}{dt}\oint_v \frac{1}{2}(\varepsilon_0 E^2 + \mu_0 H^2)dv$$

where $\vec{P} = \vec{E}\times\vec{H}$ is known as pointing vector.

Momentum and Radiation Pressure:-



Let the E.M. wave is allowed to incident upon black body of area 'A' The Pressure,

$$P = \frac{f}{A}$$

$\because$ workdone $W = f \cdot x$

and power $\phi = \frac{f \cdot x}{t} \Rightarrow f = \frac{\phi t}{x}$

$$\therefore P = \frac{\phi t}{xA}$$

$$\text{or } P = \frac{\phi}{cA} = \frac{\phi/A}{c}$$

$\Rightarrow P = \frac{E}{c}$ where E = Power per unit area is k/a irradiance.

E.M wave has a momentum moving by photon energy.

$$E = h\nu = \frac{hc}{\lambda}$$

$$\& \; P = \frac{E}{c}$$

This is known as momentum of E.M wave

—x—x—