

B.Sc Part-II (H) & Sub Paper-III Gr-B  
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ELECTRO MAGNETIC THEORY OF DISPERSION

Electromagnetic theory of dispersion: anomalous dispersion

Assumption:

- 1) There is no appreciable intersection between the atoms of gases molecules.
- 2) The electric field of the electromagnetic wave induces a dipole moment in the gas molecule.
- 3) Electrons are bound to nucleus in an atom by linear restoring force.
- 4) Over the atom or molecules  $E$  is constant.

$$E = E_0 e^{-i(\omega t - k \cdot r)}$$

$$\approx E_0 e^{-i\omega t} \quad (1)$$

The electric field force acting upon an electron in  $eE$ . The electron is also subject to elastic force  $(-kx)$  and damping force  $(-\alpha \frac{dx}{dt})$ . The eqn of motion of

electron

$$m \frac{d^2 x}{dt^2} = eE - \alpha \frac{dx}{dt} - kx$$

$$\frac{d^2 x}{dt^2} + \frac{\alpha}{m} \frac{dx}{dt} + \frac{k}{m} x = \frac{eE}{m} \quad (2)$$

where  $\gamma = \frac{\alpha}{m}$  and  $\omega_0^2 = \frac{k}{m}$

The solution of eqn<sup>n</sup> (3) is given by  

$$\delta = \frac{e/m E_0 e^{-i\omega t}}{(\omega_0^2 - \omega^2 - i\gamma\omega)} \quad \text{--- (3)}$$

The polarisation P is equal to the dipole moment (eδ) per unit volume so that if there are N dispersion electrons per unit volume then

$$P = Ne\delta = \frac{Ne^2/m E_0 e^{-i\omega t}}{(\omega_0^2 - \omega^2 - i\gamma\omega)} \quad \text{--- (4)}$$

But due to the interaction between the electron the angular velocity of different electron are differenced. Therefore the average polarisation of FK oscillates is given by

$$P = Ne \sum_k f_k \delta_k = \frac{Ne^2 E}{m} \left\langle \frac{f_k}{(\omega_k^2 - \omega^2 - i\gamma_k\omega)} \right\rangle \quad \text{--- (5)}$$

The electrical polarisation polarisability  $\epsilon$  is given by

$$\epsilon = \frac{P}{E} = \frac{Ne^2}{m} \left\langle \frac{f_k}{(\omega_k^2 - \omega^2 - i\gamma_k\omega)} \right\rangle \quad \text{--- (6)}$$

~~The electrical polarisation~~  
 The dielectric constant (K) is given by

$$K = 1 + \frac{1}{\epsilon_0} \frac{P}{E} = 1 + \frac{1}{\epsilon_0} \frac{Ne^2}{m} \left\langle \frac{f_k}{(\omega_k^2 - \omega^2 - i\gamma_k\omega)} \right\rangle \quad \text{--- (7)}$$

The refractive index is equal to the square root of dielectric constant

$$n^2 = 1 + \frac{1}{\epsilon_0} \frac{4\pi N e^2}{m} \left\langle \frac{f_k}{(\omega_k^2 - \omega^2 - i\gamma_k\omega)} \right\rangle \quad \text{--- (8)}$$

This is K/a dispersion formula.



**\* Anomalous dispersion: -**

Due to so many natural frequency of the electron the dispersion  $\vec{E}$  is disturbed. For simplicity we assume that there is one natural frequency  $\omega_p = \omega_0$

$$n^{*2} = 1 + \frac{1}{4\pi\epsilon_0} \frac{4\pi N e^2}{m} \cdot \frac{1}{\omega_0^2 - \omega^2 - i\gamma\omega}$$

$$n^{*2} = 1 + \frac{1}{4\pi\epsilon_0} \frac{2\pi N e^2}{m} \cdot \frac{1}{\omega_0^2 - \omega^2 - i\gamma\omega} \quad \text{--- (9)}$$

Multiplying numerator and denominator of right hand side by  $\omega_0^2 - \omega^2 + i\gamma\omega$  we get

$$n^{*2} = 1 + \frac{1}{4\pi\epsilon_0} \frac{2\pi N e^2}{m} \left[ \frac{\omega_0^2 - \omega^2 + i\gamma\omega}{(\omega_0 - \omega)^2 - (i\gamma\omega)^2} \right]$$

$$n^{*2} = 1 + \frac{2\pi N e^2}{4\pi\epsilon_0 m} \frac{\omega_0^2 - \omega^2}{(\omega_0 - \omega)^2 + (\gamma\omega)^2} + i \frac{2\pi N e^2 \gamma \omega}{4\pi\epsilon_0 m (\omega_0^2 - \omega^2)^2 + \gamma^2 \omega^2}$$

Now the complete refractive index

$$n^{*2} = n + iK$$

$$n = 1 + \frac{2\pi N e^2}{4\pi\epsilon_0 m} \frac{\omega_0^2 - \omega^2}{(\omega_0 - \omega)^2 + \gamma^2 \omega^2} \quad \text{--- (1)}$$

$$K = \frac{2\pi N e^2}{4\pi\epsilon_0 m} \frac{\gamma \omega}{(\omega_0 - \omega)^2 + \gamma^2 \omega^2}$$

