

LANGRANGIAN MECHANICS

1) Generalized co-ordinate :- To describe the configuration of a system we select smallest possible no. of variables k/a Generalized co-ordinates.

The Generalized co-ordinate is represented by $q_1, q_2, q_3, \dots, q_n$. If there are 'k' constraints of motion the generalized co-ordinate is reduced to $(n-k)$ for the system of n particle. The position of zth particle in the form of Generalized co-ordinate is given by

$$\begin{aligned} x_i &= x_i(q_1, q_2, q_3, \dots, q_n, t) \\ y_i &= y_i(q_1, q_2, q_3, \dots, q_n, t) \\ z_i &= z_i(q_1, q_2, q_3, \dots, q_n, t) \\ \Rightarrow \delta_i &= \delta_i(q_1, q_2, q_3, \dots, q_n, t) \end{aligned}$$

Hence a generalized co-ordinate is defined as the independent co-ordinate sufficient to specify the configuration of dynamical system

2) Principle of Virtual Work :- The total work done by the particle of system in equilibrium for small displ. δx_i due to the force F_i acting upon zth particle is equal to zero

$$\sum_i F_i \delta x_i = 0$$

3) D'Alembert's Principle :- The D'Alembert's Principle depends upon the Principle of Virtual work. The Virtual work states the total work done by the system of particle for small displacement δx_i for zth particle due to force F_i is equal to zero

$$\sum F_i \delta x_i = 0$$

According to D'Alembert's, the system of particle will be in equilibrium under the action force F_i and reactional force P_i

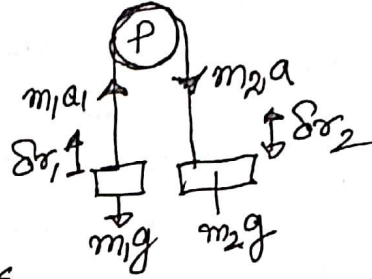
$$\sum (F_i - P_i) \delta x_i = 0$$

This is k/a D'Alembert's Principle.

Application of D'Alembert's Principle

① At Wood Machine 1 -

Let us consider At Wood machine having masses m_1 & m_2 acceleration a_1 and a_2 & their displacement in time δt are δx_1 & δx_2 respectively.



Applying D'Alembert's principle

$$(m_1 g_1 - m_1 a_1) \delta x_1 + (m_2 g - m_2 a_2) \delta x_2 = 0$$

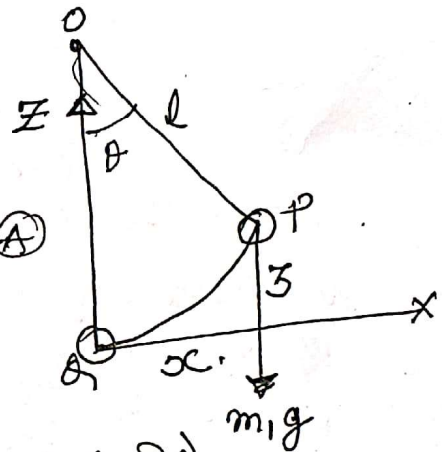
Now $\delta x_2 = -\delta x_1$ & $a_2 = -a_1$

$$\therefore (m_1 g - m_1 a_1) \delta x_1 - (m_2 g + m_2 a_1) \delta x_1 = 0$$

$$\Rightarrow \boxed{a_1 = \left(\frac{m_1 - m_2}{m_1 + m_2} \right) g}$$

② Simple pendulum :- Let us consider a simple pendulum of length l & mass of bob is m .

The vertical displacement of bob along x and z -axis are δx & δz the applied force along these axis are " X " and Z



$$(X - m\ddot{x})\delta x + (Z - m\ddot{z})\delta z = 0 \quad \text{--- (A)}$$

Now $z = -mg$ & $l^2 = z^2 + x^2$

$$\Rightarrow 0 = 2z\delta z + 2x\delta x$$

$$\Rightarrow \delta z = -\frac{x\delta x}{z}$$

From equⁿ (A), $0 = m\ddot{x}\delta x + [-mg - m\ddot{z}]\left(-\frac{x\delta x}{z}\right) = 0$

$$\text{or, } \ddot{x} = (g + \ddot{z}) \frac{x}{z}$$

Hence the accelⁿ is directly proportional to the displacement. The motion will be S.H.M. The time period of S.H.M is given by.

$$T = 2\pi \sqrt{\frac{\text{displ}}{\text{accel}^n}}$$

$$= 2\pi \sqrt{\frac{x/\ddot{x}}{g}}$$

$$\boxed{T = 2\pi \sqrt{\frac{l}{g}}}$$

$\therefore \ddot{z} = 0$ & $z = l$