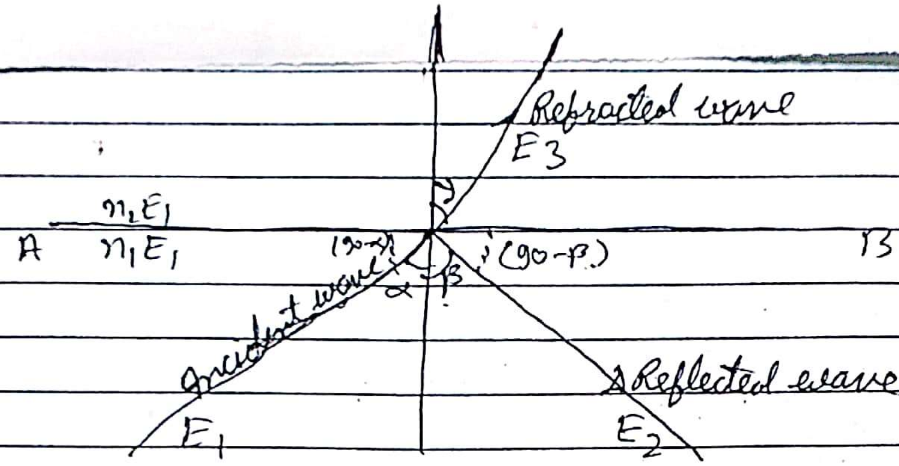
REFLECTION & REFRACTION OF ELECTRO MAGNETIC WAVE



Let us consider electromagnetic wave is allowed to incident upon AB which divide the medium of refractive indices n_1 & n_2. ~~Let~~

Let us consider ~~electromagnetic~~ the electric vector of incident ray, refracted ray and reflected ray are E_1, E_2 & E_3

Let the wave factors of ~~incden~~ incident ray & reflected ray & refracted ray are K_1, K_2 & K_3

The equ^n of incident wave, reflected wave and refracted wave are given by

$$\left.\begin{aligned} E_1 &= E_{01}\, e^{i(\omega t - K_1 r)} \\ E_2 &= E_{02}\, e^{i(\omega t - K_2 r)} \\ E_3 &= E_{03}\, e^{i(\omega t - K_3 r)} \end{aligned}\right\} \longrightarrow (A)$$

The scalar component of electric field along Y dir^n

$$E_1 y = A_1 y\, e^{i(\omega_1 t - K_1 r)}$$

$$E_2 y = A_2 y\, e^{i(\omega_2 t - K_2 r)}$$

$$E_3 y = A_3 y\, e^{i(\omega_3 t - K_3 r)}$$

where A_1, A_2 & A_3 are the amplitude along incident ray, reflected ray & refracted ray.

Now $E_1y + E_2y = E_3y$

$$A_1y e^{i(\omega_1 t - k_1 r)} + A_2 y e^{i(\omega_2 t - k_2 r)} = A_3 y e^{i(\omega_3 t - k_3 r)} \quad \text{—(C)}$$

Differentiating equ^n (3) w.r to time we get.

$$A_1 y i\omega_1 e^{i(\omega_1 t - k_1 r)} + A_2 y i\omega_2 e^{i(\omega_2 t - k_2 r)} = A_3 y e\omega_3^{i(\omega_3 t - k_3 r)} \quad \text{—(D)}$$

On solving equ^n (C & D).

$$A_1 y e^{i(\omega_1 t - k_1 r)} (\omega_3 - \omega_1) = A_2 y e^{i(\omega_2 t - k_2 r)} (\omega_2 - \omega_3) \quad \text{—(E)}$$

This equ^n is valid for $\omega_1 = \omega_2 = \omega_3$

From equ^n (C) we get.

$$A_1 y e^{-ik_1 r} + A_2 y e^{-ik_2 r} = A_3 y e^{-ik_3 r} \quad \text{—(F)}$$

Differentiating it w.r to r we get.

$$A_1 y i k e^{-ik_1 r} + A_2 y i k_2 e^{-ik_2 r} = A_3 y i k_3 e^{-ik_3 r} \quad \text{—(G)}$$

Taking dot product with $\vec{r}$ w.r.t.

$$\vec{k_1}\cdot\vec{r} A_1 y e^{-ik_1 r} + \vec{k_2}\cdot\vec{r} A_2 y e^{-ik_2 r} = \vec{k_3}\cdot\vec{r} A_3 y e^{-ik_3 r}$$

On solving ~~we~~ equ^n (G) & H we get.

$$A_1 y e^{-ik_1 r}(\vec{k_3}\cdot\vec{r} - \vec{k_1}\cdot\vec{r}) = A_2 y e^{-ik_2 r}(\vec{k_2}\cdot\vec{r} - \vec{k_3}\cdot\vec{r})$$

This equ^n is valid for

$$\vec{k_1}\cdot\vec{r} = k_1 r \cos(90-\alpha) = k_1 r \sin\alpha$$

$$\vec{k_2}\cdot\vec{r} = k_2 r \sin\beta$$

$$\vec{k_3}\cdot\vec{r} = k_3 r \sin\gamma$$

From equ^n (I) for reflection

$$\cancel{k_1} \; k_1 = k_2$$

$$\boxed{\alpha = \beta}$$

This is law of reflection

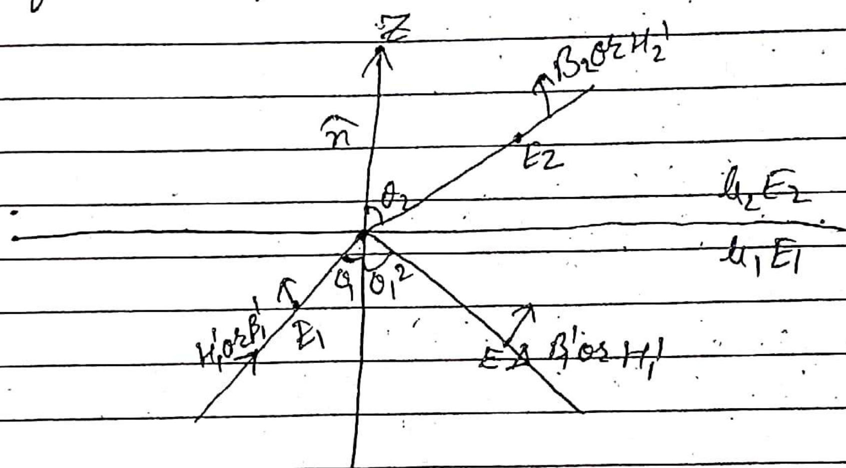
Again, let the velocity of wave for incident ray and refracted ray are v_1 & v_2

Using equ^n
$k_1 r \sin\alpha = k_2 r \sin\gamma$

$$\frac{\sin\alpha}{\sin\gamma} = \frac{k_2}{k_1} = \frac{v_1}{v_2} = \frac{n_2}{n_1}$$

$\Rightarrow n_1 \sin\alpha = n_2 \sin\gamma$
This is is Snell's law of refraction

- ☆ Fresnel equ^n of reflection and refraction
The equ^n relating the amplitude of reflected and transmitted wave with those of incident wave are k/a fresnel's equ^n.



Let the electric vector E_1 and magnetic vector B_1 are $\perp^r$ upon the direction of propogation constant k_1
The equ^n of continuity of electric vector along the dir^n of reflected surface

$$EO_1 + EO_1' = EO_2 \quad \text{—— (1)}$$

Similarly the magnetic vector continuity along the Z axis

$$H_1 t + H_1' t = H_2 t$$
$$HO_1 \cos\theta + HO_1' \cos\theta_1' = -HO_2 \cos\theta_2$$
$$(HO_1 - HO_1') \cos\theta_1 = HO_2 \cos\theta_2 \quad \text{—— (2)}$$

$$B_1 = \frac{k_1 \times E_1}{\omega_1} \Rightarrow H_1 = \frac{k_1 \times E_1}{\mu_1 \omega_1}$$

Or $H_1 = \dfrac{K_1 \hat{n}_1 \times E_1}{\mu_1 \omega_1}$

$K_1 = \dfrac{\omega_1}{V_1} = \sqrt{\varepsilon_1 \mu_1}\, \omega_1$

$\cancel{H_1 = \dfrac{\omega_1 \hat{n}_1 \times E_1}{V_1}}$

$H_1 = \dfrac{\sqrt{\varepsilon_1 \mu_1}\, \hat{n}_1 \times E_1}{\mu_1}$

$H_1 = \sqrt{\dfrac{\varepsilon_1}{\mu_1}}\, \hat{n}_1 E_1$

$$\left.\begin{aligned} H_{01} &= \sqrt{\frac{\varepsilon_1}{\mu_1}}\, E_{01} \\ H'_{01} &= \sqrt{\frac{\varepsilon_1}{\mu_1}}\, E'_{02} \\ H_{02} &= \sqrt{\frac{\varepsilon_2}{\mu_2}}\, E_{02} \end{aligned}\right\} \text{—— (3)}$$

From eqⁿ (2)

$$\sqrt{\frac{\varepsilon_1}{\mu_1}}\left(E_{01} - E'_{01}\right) \cos\theta_1 = \sqrt{\frac{\varepsilon_2}{\mu_2}}\, E_{02} \cos\theta_2$$

using eqⁿ (1)

$$\sqrt{\frac{\varepsilon_1}{\mu_1}}\left(E_{01} - E'_{01}\right) \cos\theta_1 = \sqrt{\frac{\varepsilon_2}{\mu_2}}\left(E_{01} + E'_{01}\right) \cos\theta_2$$

On solving we get

$$\frac{E_{01} - E'_{01}}{E_{01} + E'_{0}} = \frac{\sqrt{\varepsilon_2/\mu_2}\cdot\cos\theta_2}{\sqrt{\varepsilon_1/\mu_1}\cos\theta_1}$$

$$\frac{2E_{01}}{2E_{02}} = \frac{\sqrt{\varepsilon_1/\mu_1}\cos\theta_1 + \sqrt{\varepsilon_2/\mu_2}\cos\theta_2}{\sqrt{\varepsilon_1/\mu_1}\cos\theta - \sqrt{\varepsilon_2/\mu_2}\cos\theta_2}$$

Similarly eleminating E'_{01} from (2) & (4)

$$\sqrt{\frac{\varepsilon_1}{\mu_1}}\left[E_{01} - E_{02} + E_{01}\right] \cos\theta_1 = \sqrt{\frac{\varepsilon_2}{\mu_2}}\, E_{02} \cos\theta_2$$

$$2E_{01} \sqrt{\frac{\varepsilon_1}{\mu_1}} \cos\theta_1 = E_{02}\left[\sqrt{\frac{\varepsilon_2}{\mu_2}} \cos\theta_2 + \sqrt{\frac{\varepsilon_1}{\mu_1}} \cos\theta_1\right]$$

$$\frac{E_{01}}{E_{02}} = \frac{\sqrt{\varepsilon_2/\mu_2}\cos\theta_2 + \sqrt{\varepsilon_1/\mu_1}\cos\theta_1}{2\sqrt{\varepsilon_1/\mu_1}\cos\theta_1} \quad \text{--- (6)}$$

These equ^n (5) & (6) are k/a Fresnels equ^n for nonconducting medium.

$\therefore \mu_1 = \mu_2 = \mu_0$

then the refractive index of the medium

$n_1 = \sqrt{\varepsilon_1/\varepsilon_0}$

& $n_2 = \sqrt{\varepsilon_2/\varepsilon_0}$

$$\frac{n_2}{n_1} = \sqrt{\frac{\varepsilon_2}{\varepsilon_1}} \quad \text{--- (7)}$$

From equ^n (5) $\sqrt{\frac{\varepsilon_1}{\mu_0}}\cos\theta_1 + \sqrt{\frac{\varepsilon_2}{\mu_0}}\cos\theta_2$

$$\Rightarrow \frac{\cos\theta_1 + \sqrt{\varepsilon_2/\varepsilon_1}\cos\theta_2}{\cos\theta_1 - \sqrt{\varepsilon_2/\varepsilon_1}\cos\theta_2}$$

$$\frac{E_{01}}{E_{02}} = \frac{\cos\theta_1 + n_2/n_1\cos\theta_1}{\cos\theta_1 - n_2/n_1\cos\theta_2} \quad \text{--- (8)}$$

From equ^n (6) for refraction

$$\frac{E_{01}}{E_{02}} = \frac{\sqrt{\varepsilon_2/\mu_0}\cos\theta_2 + \sqrt{\varepsilon_2/\mu_2}\cos\theta_1}{2\sqrt{\varepsilon_1/\mu_0}\cos\theta_1}$$

$$= \frac{\sqrt{\varepsilon_2/\varepsilon_1}\cos\theta_2 + \cos\theta_1}{2\cos\theta_1}$$

$$\frac{E_{01}}{E_{02}} = \frac{n_2/n_1\cos\theta_2 + \cos\theta_1}{2\cos\theta_1} \quad \text{--- (9)}$$

using snell's law $\frac{n_2}{n_1} = \frac{\sin\theta_1}{\sin\theta_2}$

$$\frac{E_{O_1}}{E_{O_2}} = \frac{\frac{\sin\theta_1}{\sin\theta_2}\cos\theta_2 + \cos\theta_1}{2\cos\theta_1 \sin\theta_2}$$

$$\frac{E_{O_1}}{E_{O_2}} = \frac{\sin(\theta_1+\theta_2)}{2\cos\theta_1 \sin\theta_2} \quad \text{——— (10)}$$

For reflection

$$\frac{E_{O_1}}{E_{O_2}} = \frac{\cos\theta_1 + \frac{\sin\theta_1}{\sin\theta_2}\cos\theta_2}{\cos\theta_1 - \frac{\sin\theta_1}{\sin\theta_2}\cos\theta_2}$$

$$\frac{E_{O_1}}{E_{O_2}} = \frac{\sin(\theta_1+\theta_2)}{\sin(\theta_2-\theta_1)} \quad \text{——— (11)}$$

Equation (10) & (11) gives the fresnel's eqn for noncondueting medium

☆ Scattering of E.M wave by free electron :-
(Thomson scattering)

When electromagnetic wave is allowed to incident on a system of charge particle. The charge parellel of the oscillates and they absorb incident wave and will be ~~emitted~~ reemitted in the space in all direction. This phenomenon is K/a scattering.

Let us consider an electron of mass 'm' and charge 'e' is kept in a polarished light.

let the incident electromagnetic wave having intensity along x-dirn and moving along z dirn. The force eE accellerates the electron by accelleration d^2x/dt^2

$$m \cdot \frac{d^2x}{dt^2} = eE = eE_0 e^{-i(\omega t - kx)} \quad \text{——— (A)}$$