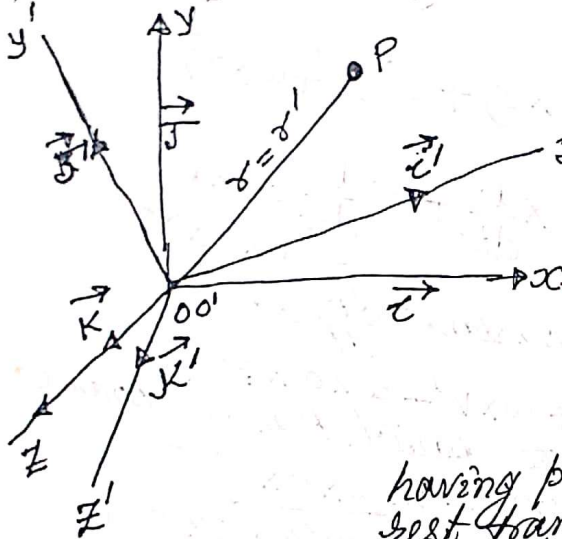


ROTATING FRAME OF REFERENCE

Coriolis force :- when a body is moving in a rotating frame of reference, a Pseudo force is developed which deflects the body perpendicular to the path k/a .
 Coriolis's force.

Expression of Coriolis's force :-



Let us consider two frame of reference in which (x, y, z) is rest frame and x', y', z' is rotating frame. The Unit vector of this frame are $(\hat{i}, \hat{j}, \hat{k})$ and $(\hat{i}', \hat{j}', \hat{k}')$ respectively.

Let us consider a particle P having position vectors \vec{r} and \vec{r}' in rest frame and rotating frame respectively

$$\vec{r} = \hat{i}x + \hat{j}y + \hat{k}z \quad \text{--- (A)}$$

$$\& \vec{r}' = \hat{i}'x' + \hat{j}'y' + \hat{k}'z'$$

$$\text{Now, } \left(\frac{d\vec{r}}{dt}\right)_{\text{fix}} = \hat{i} \frac{dx}{dt} + \hat{j} \frac{dy}{dt} + \hat{k} \frac{dz}{dt}$$

$$\text{Again } \left(\frac{d\vec{r}}{dt}\right)_{\text{fix}} = \left(\frac{d\vec{r}'}{dt}\right)_{\text{rot}} = \hat{i}' \frac{dx'}{dt} + \hat{j}' \frac{dy'}{dt} + \hat{k}' \frac{dz'}{dt} + x' \frac{d\hat{i}'}{dt} + y' \frac{d\hat{j}'}{dt} + z' \frac{d\hat{k}'}{dt}$$

$$\text{Now } x' \frac{d\hat{i}'}{dt} = (\omega \times \hat{i}')x', \quad y' \frac{d\hat{j}'}{dt} = (\omega \times \hat{j}')y'$$

$$\& z' \frac{d\hat{k}'}{dt} = (\omega \times \hat{k}')z'$$

$$\therefore \left(\frac{d\vec{r}}{dt}\right)_{\text{fix}} = \left(\frac{d\vec{r}'}{dt}\right)_{\text{rot}} + x'(\omega \times \hat{i}') + y'(\omega \times \hat{j}') + z'(\omega \times \hat{k}')$$

$$\text{or, } \left(\frac{d\vec{r}}{dt}\right)_{\text{fix}} = \left(\frac{d\vec{r}'}{dt}\right)_{\text{rot}} + \vec{\omega} \times \vec{r}' \quad \text{--- (B)}$$

or, $V = V' + \omega \times r$ ————— (c)

The equation (c) gives a vector operation

$$\left(\frac{d-}{dt}\right)_{fix} = \left(\frac{d-}{dt}\right)_{rot} + \vec{\omega} \times \dots$$

Putting the velocity V in place of 'r'

$$\left(\frac{dV}{dt}\right)_{fix} = \left(\frac{dV}{dt}\right)_{rot} + \omega \times V$$

or, $\left(\frac{dV}{dt}\right)_{fix} = \frac{d}{dt}(V' + \omega \times r) + \omega \times (V' + \omega \times r)$

or, $\left(\frac{dV}{dt}\right)_{fix} = \left(\frac{dV'}{dt}\right)_{rot} + \omega \times \frac{dr}{dt} + r \cdot \frac{d\omega}{dt} + \omega \times V' + \omega \times \omega \times r$

Now ω is constant $\frac{d\omega}{dt} = 0$

$\therefore a = a' + \omega \times V' + \omega \times V' + \omega \times \omega \times r$
 the force acting on the particle P in S-frame

or, $F' = F + 2m(\omega \times V') + m(\omega \times \omega \times r)$ ————— (d)

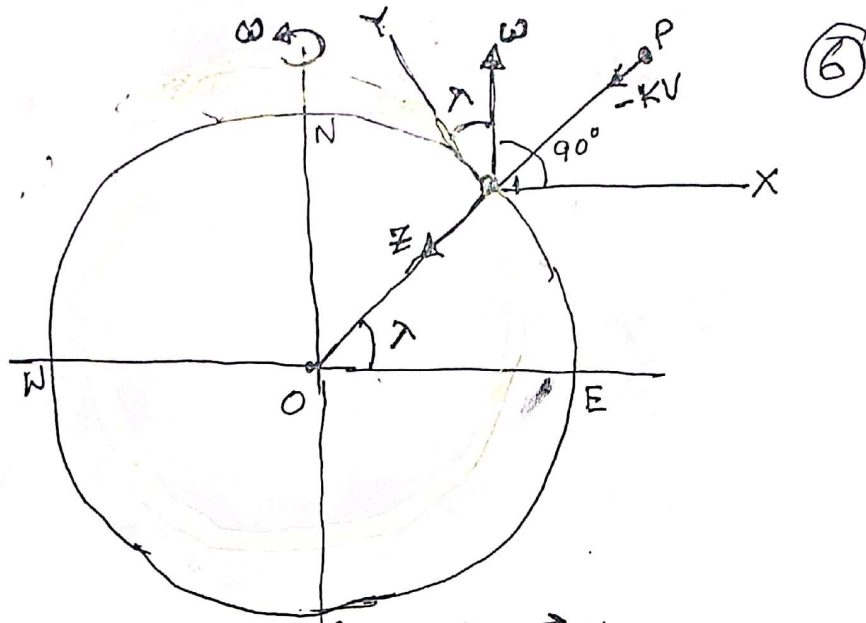
The term $m(\omega \times \omega \times r)$ is well known force K/a centrifugal force. The force $2m(\omega \times V')$ is K/a Coriolis force. The direction of this force is perpendicular to the plane passing through $(\omega \times V')$

SHIFTING OF FREELY Falling body:-

Let us consider a body of mass m is dropped from the height h at longitude λ . The angular velocity of earth is given by.

$$\omega = \vec{i} \omega_x + \vec{j} \omega_y + \vec{k} \omega_z$$

$$\omega = 0 + \vec{j} \omega_y + \vec{k} \omega_z$$



The velocity along z-axis, $\vec{V} = -K\vec{V}$
 The Coriolis force $F = -2m(\vec{\omega} \times \vec{V})$
 $= 2m[\vec{\omega} \cos \lambda + K\vec{\omega} \sin \lambda](K\vec{V})$

Now putting $V = 0 + gt$
 $\therefore F = m \frac{d^2x}{dt^2} = 2m\omega \cos \lambda gt$

This shows that body is shifted towards east

$\therefore \frac{d^2x}{dt^2} = 2\omega \cos \lambda gt$
 on double integrating we get
 $x = 2\omega \cos \lambda \frac{gt^3}{2 \times 3}$

$$x = \frac{\omega g t^3}{3} \cos \lambda$$

Again $h = 0 + \frac{1}{2}gt^2 \Rightarrow t = \sqrt{\frac{2h}{g}}$

$$\therefore x = \frac{\omega g \cos \lambda}{3} \sqrt{\frac{8h^3}{g^3}}$$

$$x = \frac{\omega \cos \lambda}{3} \sqrt{\frac{8h^3}{g}}$$

