

SCHRODINGER WAVE EQUATION OF FREE PARTICLE

The Schrodinger equation of the particle is given by

$$\nabla^2 \psi + \frac{2m}{\hbar^2} (E - V) \psi = 0 \quad \text{--- (1)}$$

For free particle $V = 0$;

$$\therefore \frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} + \frac{\partial^2 \psi}{\partial z^2} + \frac{2m}{\hbar^2} E \psi = 0 \quad \text{--- (2)}$$

Let solution of equⁿ is given by

$$\psi(x, y, z) = X_x * Y_y * Z_z = XYZ \quad \text{--- (3)}$$

Where X_x, Y_y, Z_z are the functions along x, y and z -axes.

Now $\frac{\partial^2 \psi}{\partial x^2} = \frac{\partial^2 X}{\partial x^2} YZ$ & $\frac{\partial^2 \psi}{\partial x^2} = \frac{\partial^2 X}{\partial x^2} YZ$

and $\frac{\partial^2 \psi}{\partial y^2} = XZ \frac{\partial^2 Y}{\partial y^2}$ and $\frac{\partial^2 \psi}{\partial z^2} = XY \frac{\partial^2 Z}{\partial z^2}$

From equⁿ (2)

$$YZ \frac{\partial^2 X}{\partial x^2} + ZX \frac{\partial^2 Y}{\partial y^2} + XY \frac{\partial^2 Z}{\partial z^2} + \frac{2m}{\hbar^2} E XYZ = 0$$

Dividing by XYZ

$$\frac{1}{X} \frac{\partial^2 X}{\partial x^2} + \frac{1}{Y} \frac{\partial^2 Y}{\partial y^2} + \frac{1}{Z} \frac{\partial^2 Z}{\partial z^2} + \frac{2m}{\hbar^2} E = 0 \quad \text{--- (4)}$$

In this equⁿ each term is a constant quantity

$$\therefore \frac{1}{X} \frac{\partial^2 X}{\partial x^2} = K_x, \quad \frac{1}{Y} \frac{\partial^2 Y}{\partial y^2} = K_y$$

$$\& \frac{1}{Z} \frac{\partial^2 Z}{\partial z^2} = K_z$$

Equⁿ (4) becomes

$$K_x + K_y + K_z = \frac{2mE}{\hbar^2}$$

Let us put

$$K_x = \frac{2mE_x}{\hbar^2}, K_y = \frac{2mE_y}{\hbar^2} \text{ \& } K_z = \frac{2mE_z}{\hbar^2}$$

$$\therefore \frac{1}{X} \frac{\partial^2 X}{\partial x^2} = -\frac{2mE_x}{\hbar^2}$$

$$\text{or, } \frac{\partial^2 X}{\partial x^2} + \frac{2mE_x}{\hbar^2} X = 0 \quad \text{--- (5)}$$

the general solution of this equation is

$$X(x) = N_x \sin \left[\frac{1}{\hbar} \sqrt{2mE_x} (x - x_0) \right] \quad \text{--- (6)}$$

$$\text{Similarly } Y(y) = N_y \sin \left[\frac{1}{\hbar} \sqrt{2mE_y} (y - y_0) \right]$$

$$\text{ \& } Z(z) = N_z \sin \left[\frac{1}{\hbar} \sqrt{2mE_z} (z - z_0) \right]$$

$$\text{Now, } \psi(x, y, z) = N_x \sin \left[\frac{1}{\hbar} \sqrt{2mE_x} (x - x_0) \right] \sin \left[\frac{1}{\hbar} \sqrt{2mE_y} (y - y_0) \right] \sin \left[\frac{1}{\hbar} \sqrt{2mE_z} (z - z_0) \right] \quad \text{--- (7)}$$

where $N = N_x \cdot N_y \cdot N_z =$ Normalisation constant for the physical interpretation of wave function
let $E_y = E_z = 0$ \& $E_x = E$

$$\therefore \psi(x, y, z) = N \sin \left[\frac{1}{\hbar} \sqrt{2mE} (x - x_0) \right] \quad \text{--- (8)}$$

For the time dependent wave function

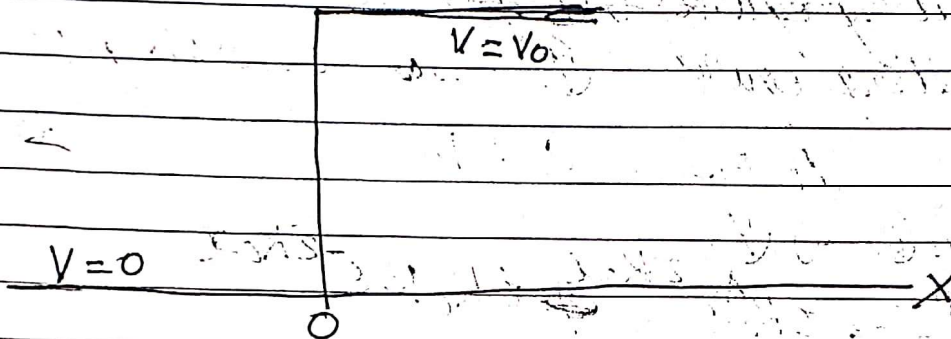
$$\psi(x, t) = N \sin \frac{1}{\hbar} \sqrt{2mE} (x - x_0) e^{iEt/\hbar} \quad \text{--- (9)}$$

This represents a set of standing waves with wavefront-normal to the x -axis. The wavelength is given by

$$\lambda = \frac{h}{\sqrt{2mE}}$$

Where $E = \frac{1}{2}mv^2 = \text{Energy Eigen Value}$.

SCHRODINGER'S WAVE EQUATION for POTENTIAL STEP



For Potential Step, $V(x) = 0$ for $x < 0$
 $V(x) = V_0$ for $x > 0$

The time independent S.E is given by

$$\frac{d^2\psi}{dx^2} + \frac{2m}{\hbar^2}(E - V)\psi = 0 \quad (1)$$

$$\text{for } x < 0, \quad \frac{d^2\psi}{dx^2} + \frac{2mE}{\hbar^2}\psi = 0 \quad (2)$$

$$\text{for } x > 0, \quad \frac{d^2\psi}{dx^2} + \frac{2m(E - V_0)}{\hbar^2}\psi = 0 \quad (3)$$

$$\text{or, for } x < 0 \quad \frac{d^2\psi}{dx^2} + K_0^2\psi = 0 \quad (4)$$

$$\text{and for } x > 0 \quad \frac{d^2\psi}{dx^2} + K^2\psi = 0 \quad (5)$$

$$\text{Where } K_0 = \sqrt{2mE}/\hbar \quad \& \quad K = \sqrt{2m(E - V_0)}/\hbar$$