HEISENBERG'S UNCERTAINTY PRINCIPLE

Statement :- The exact position and momentum of a particle cannot be determined simultaneously with desired accuracy.

Let Δx be the error in position and ΔP be the error in determining the momentum of the particle at the same instant. These quantities are related as

$$\Delta x \cdot \Delta P \geq \frac{h}{4\pi} = \frac{\hbar}{2} \quad \text{--- (1)}$$

$\hbar = \frac{h}{2\pi}$

Multiplying and dividing by v (velocity of the particle)

We get

$$\frac{\Delta x}{v} \cdot \Delta P . v \geq \hbar/2$$

$$\text{or,} \quad \Delta t \cdot \Delta E \geq \hbar/2 \quad \text{--- (2)}$$

Where ΔE represents the uncertainty in measurement of energy and Δt the uncertainty in measurement of time.

In equ^n (1) & (2), if one quantity is measured accurately, the other quantity less accurate.

Proof :- The group velocity of de-Broglie waves associated with moving particle is

$$v_g = \frac{d\omega}{dK} = \frac{d\omega}{d\frac{2\pi}{\lambda}} = \frac{-\lambda^2}{2\pi}\frac{d\omega}{d\lambda} \quad \text{--- (3)}$$

$$\because \lambda = \frac{h}{p} \quad \therefore v_g \ldots$$

$$\therefore d\lambda = \frac{-h}{p^2} dp \quad \text{--- (4)}$$

from (3) & (4)

$$v_g = \frac{-h^2 \, d\omega}{p^2 2\pi (-h/p^2 \, dp)} = \frac{h}{2\pi} \cdot \frac{d\omega}{dp} \quad \text{--- (5)}$$

The ~~the~~ group of waves represents a particle moving with velocity v along x-axis then.

$$v = \frac{\Delta x}{\Delta t}$$

or, $v_g = \frac{\Delta x}{\Delta t} = \frac{h}{2\pi}\frac{d\omega}{dp}$

$$\Rightarrow \Delta p \cdot \Delta x = \frac{h}{2\pi}\Delta\omega \cdot \Delta t \quad \text{—— (6)}$$

If the angular velocity of the wave is to be measured, the least time of measurement will be the time required for one complete wave length to pass a reference point. If Δt is this time then,

$$\Delta t \geq \frac{1}{\Delta\omega} \quad \text{i.e. } \Delta\omega \cdot \Delta t \geq 1$$

From equ^n(6) $\Delta p \cdot \Delta x \geq \frac{h}{2\pi}$

This is uncertainty principle. This relation shows that it is impossible to determine simultaneously both the position and momentum of the particle accurately.

Illustration (example) [Experiment]:

Determination of position of particle by a Microscope:

Let O be the electron, the λ is the wavelength of γ-rays and α the semi-vertical angle of the cone of ray. The resolving power of Microscope

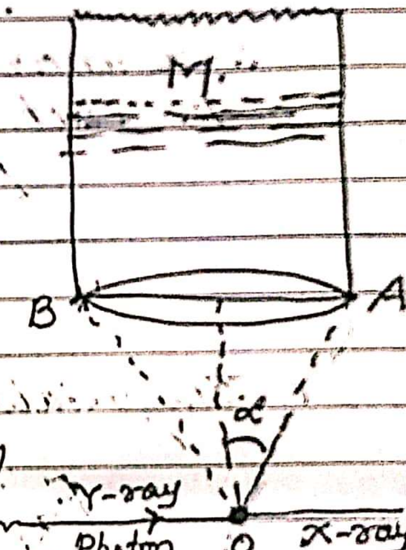
$$\Delta x = \frac{\lambda}{2\sin\alpha}$$

where Δx is the shortest distance between two points which is the uncertainty in the measurement of the position of electron.

The electron is seen by the γ-radiation scattered by it into the microscope. The momentum of scattered photon is $\frac{h}{\lambda}$

The component of momentum along OA

$$= \frac{h}{\lambda} \sin\alpha.$$



Momentum imparted to the electron

$$= \frac{h}{\lambda'} - \frac{h}{\lambda} \sin\alpha.$$

Where λ' is the wavelength of the γ-radiation.

$\lambda = \frac{h}{p}$

Momentum imparted to the electron due to the scattering towards OB

$$= \frac{h}{\lambda'} - \left[-\frac{h}{\lambda} \sin\alpha\right]$$

$$= \frac{h}{\lambda'} + \frac{h}{\lambda} \sin\alpha.$$

The uncertainty of the error involved in the measurement of momentum of electron

$$\Delta P_x = \left(\frac{h}{\lambda'} + \frac{h}{\lambda} \sin\alpha\right) - \left(\frac{h}{\lambda'} - \frac{h}{\lambda} \sin\alpha\right)$$

$$\Delta P_x = \frac{2h}{\lambda} \sin\alpha$$

$$\therefore \Delta P_x \cdot \Delta x = \frac{\lambda}{2\sin\alpha} \cdot \frac{2h}{\lambda} \sin\alpha = h$$

$$\Delta P_x \Delta x \geq h/2\pi$$

This is the agreement with the uncertainty relation. This shows that when the position of the electron is measured more accurately, the uncertainty of momentum becomes larger.

Electrons cannot exist in the nucleus

Nuclear radius is of the order of 10^{-14}m, hence its diameter is of the order of 2×10^{-14}m. The uncertainty of measurement of position of electron inside the nucleus

$$\therefore \Delta x = 2\times10^{-14}m$$

$\therefore$ The uncertainty of measurement of momentum of the electron will be

$$\Delta p = \frac{h}{\Delta x} = \frac{6.62\times10^{-34}}{2\times10^{-14}}$$

$$= 3.31\times10^{-20}\ kg\ m/s$$

The relativistic energy of the electron

$$E^2 = P^2C^2 + m_0^2C^4$$

Now $m_0^2c^4$ may neglected in compression to P^2c^2

$$\therefore E = PC = 3.31\times10^{-20}\times3\times10^8 J$$

$$= 62\times10^6 eV = 62 MeV$$

$\frac{620}{1.6}$ $=4$

Thus if the electron resides in the nucleus it should have an energy of the order of 62×10^6MeV. However, electron emitted during B-decay have energy of the order of 3 MeV. Hence we conclude that electron cannot exist in the nucleus.